A

TREATISE

OF THE

SPHERE;

SHEWING

How it is deriv'd from that Theory which justly Asserts the

MOTION of the EARTH:

As also of the Projections of it, both

***Orthographical* and *Stereographical*;**

DEMONSTRATING

Their Properties from Fundamental *Propositions*, and shewing their Uses. With the Resolution of *Astronomical* and *Chorographical* PROBLEMS.

By the late Reverend JOHN WITTY M. A. and Chaplain to his Grace the Duke of *Devonshire*.

The SECOND EDITION.

Revis'd and Improv'd by JAMES HODGSON, Master of the *Royal Mathematical School*, and *Fellow* of the ROYAL SOCIETY.

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THE PREFACE

THE *Author* of the following *Treatise*, having design'd it for the Use of a young Nobleman of the first Quality, it may be requisite to give the Reader some Account of Mr. *Witty*, and what he proposed to himself in writing it.

His Genius leading him early to Studies of this Nature, he was retain'd by Mr. *Flamstead* in the Year 1705, which gave him an Opportunity of acquainting himself perfectly with the Calculation of *Lunar*, and the Construction of *Solar* Eclipses according to the Method taught in the Doctrine of the *Sphere*, (a Book wrote by Mr *Flamstead* in the Year 1680, and published with the Posthumous Works of Sir *Jonas Moor*) with the Ground-work Theorems of *Stereographical Projections*; of which little as yet had been published in the *English* Tongue.

Clavius, indeed, and *Aquilonius* have wrote well and largely of *Projections* of the *Sphere*; but their Works are Voluminous, and contain a great deal more than is necessary for the understanding of them. The Doctrine of the *Sphere*, that supposed the Reader to understand the Ground-work *Theorems* was now become very scarce; all the Editions of the Book, with which it was printed, being sold off, and much enquired after. Mr. *Witty* therefore proposed to himself to write a new Treatise on the same Subject, that might

comprehend all the *Theorems* neceſſary for any ſort of *Projections* of the *Sphere*, and dividing any of the projected *Circles*, and to ſhew both how the Old and New *Projections* are derived from that Ancient and True *Theory* which ſuppoſes the *Motion of the Earth*, and where the Triangles are form'd in both, for the Reſolution of all *Aſtronomical Problems*. This ſo far as he has proceeded is done in a Method altogether Mathematical, premiſing, firſt, ſuch Definitions as his Subject requires, then *Axioms*, *Theorems*, *Propoſitions* and *Corollaries* reſulting from them, with *Problems*; all which he Demonſtrates plainly and fully, with this farther Advantage to the Readers that his Language is always pure, his Expreſſions proper, and his Thoughts clear; for he was Maſter of his Subject and was able to explain it fully and clearly to others: Qualifications not always to be met with in Authors.

During his Reſidence at the *Obſervatory Royal*, by Aſſiſting Mr. *Flamſtead* in his Aſtronomical Calculations; he became fully ſkill'd in all the Methods uſed of deducing the Places of the *Fix'd Stars* and *Planets* from their obſerved Meridional Diſtances from the *Vertex*, and Differences of the Times of their *Tranſits* over the *Meridian*; as alſo of deducing their *Longitudes* and *Latitudes* from their determin'd *Right Aſcenſions* and *Diſtances* from the *Northern Pole*, by a New Method of Proſtapheriſes; which with the Help of ſome Subſidiary Tables prepared aforehand, is done by the Reſolution of only one *Right-angled Spherical Triangle*; whereas the old Method required the Reſolution of an *Oblique angled one*, wherein there was much more Trouble and Danger of Error: The Reader will therefore pardon him, for having

ing been a little prolix, sometimes, in his Discourses, since there are but few People that are acquainted with these Methods. There is nothing to be met with of them in other Books, which renders this very useful to our Young Students, for whose Information it is principally intended.

It may not be amiss to add, That the Author was well accomplish'd in other Parts of Learning, and much accustomed to the most strict and abstracted Reasoning. His great Acuteness and Penetration, together with a sound Judgment, rendered him very fit to appear in the Defence of the Principles of Natural and Revealed Religion, as he did successfully against those who pretended to attack it on the principles of Reason. Few People, indeed, are proper Judges of Performances of this Kind; however, I shall take this Occasion to mention two very valuable Pieces, which were publish'd by him in his Life-time, in one of which he made very good Use of Mr. *Lock*'s Principles against the *Free-Thinkers*, entitled, *The Principles of Modern Deism confuted*: The other which was generally esteem'd, he call'd, *The Reasonableness of assenting to the Mysteries of Christianity asserted and vindicated*; in which by the Way, is contained a solid *Answer* to the *Essay concerning the Use of Reason in Propositions, the Evidence whereof depends upon Human Testimony*, which at that Time made a great Noise in the World. He wrote also, when very Young, *An Essay towards a Vindication of the* Mosaick *History of the Creation of the World, and the Fall of Man; in several Letters.*

He died in the Year 1714, and in the 31*st* Year of his Age, after a few Days Illness, leaving some Sheets of this Book printed off, but the Work

Work being not complete (for he intended that it should consist of several parts, in the Course of which he proposed to treat of the Doctrine of *Eclipses*, of *Sciography*, and some other useful Matters) at the Request of the Publisher and out of respect to my deceased Friend, I undertook to finish this Part, and usher it into the World.

In the present Edition I have taken the Liberty to alter some of the Demonstrations, to insert such things as I thought might be useful, and to send it abroad as correct as possible.

From the Royal
Mathematical
School, *June* the
17*th*, 1734.

JAMES HODGSON.

THE

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INTRODUCTION.

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DEFINITION I.

If we imagine a Semi-circle to turn round upon its Diameter, that Diameter being kept, during the Circumvolution of the Semi-circle, in the same Place; by this Circumvolution upon this Diameter, as an Immoveable Axis, it will generate what we call a Sphere. Def. 1.

Scholium. THE Properties, either Absolute or Relative, of a *Sphere*, as it is a *Solid* thus generated we are not oblig'd to consider in this Place; we having only to do in the Sequel of this Discourse with the *Circles* which are usually described upon it, and which are made use of in the Solution of *Geographical*, *Astronomical*, *Sciotherical*, and other like *Problems*: We may only take notice, that from this Generation of the *Sphere*, it follows, that as all the Points of the Circumference of the generating Semi-circle are equidistant from its Centre, all the Points of the Surface of the generated *Sphere* must be so too; having all of 'em a *Radius* of the generating Semi-circle for the Measure of that Distance: And as every Point in the Circumference of the generating Semi-circle, in its Circumvolution upon its *Axis*, marks out a Circle in the Surface of the

the *Sphere*, thoſe Circles are greater or leſs, as they are more remote from, or nearer to the Extremities of the *Axis*; that which is the moſt remote being the greateſt, as having the Semidiameter of the generating Semi-circle, at Right Angles to the *Axis* it turns upon, for its *Radius*; and the reſt on each ſide decreaſing as the *Sines* of their Diſtances from the Circle generated by that *Radius* increaſe.

Def. 2. *That Circle which is form'd by the Circumvolution of the* Radius *of the generating Semi-circle at Right Angles to the* Axis *it turns upon, ſtanding upon its Centre, is call'd a* Great Circle *of the* Sphere*: All the reſt which are parallel to it are* Leſſer *Circles.*

Corollary 1. FRom this Definition it follows, that all the *great* Circles of the *Sphere* cut it into two equal Parts: As they paſs thro' its Centre, and therein give the perpendicular Height of all the Sections of the *Sphere* which they can make, the ſame, it being meaſur'd by a *Radius* of the *Sphere*: And as they are all form'd by equal *Radii*, and conſequently make the *Areas* of the *Baſes* of all their Sections equal. And whatever two Solids of the ſame kind have the *Areas* of their *Baſes*, and their Heights equal, are themſelves equal. And as *Leſſer* Circles do not paſs thro' the Centre of the *Sphere*, they muſt cut it into Unequal Parts.

Corol. 2. Hence it is alſo plain, that all *great* Circles of the *Sphere* muſt cut each other at a Semicircular Diſtance: For as they all have for their common Centre that Centre of the *Sphere*, and conſequently interſect each other in the Centre; and as the common Interſection of the Planes of any two

two of 'em is a Right-Line continued from the Centre both Ways to the Surface of the *Sphere*; hence 'tis manifest that the Intersection of these Planes is a Diameter of the *Sphere*: But it is also a Diameter of both these Circles at the same time, as it passes thro' their common Centre both ways to their respective Circumferences; therefore it cuts them both in halves, as the Diameters do all Circles: Therefore from one Point of Intersection to the other in both Circles, on each Side, is a Semicircle. Or, which is the same thing, they cut each other at a Semi-circular Distance.

Corol. 3. And if we conceive Two *great* Circles of the *Sphere* any way projected upon a Plane; their Representatives upon that Plane must evidently intersect each other at a Semi-circular Distance also: The Arches intercepted 'twixt the Points of Intersection in the Circles themselves being necessarily represented by those intercepted 'twixt the projected Points of Intersection in the Plane of the Projection: And consequently as the *great* Circles of the *Sphere* cut each other at a Semicircular Distance, their Representatives in the Plane must do the same.

The Two Extremities of the Fix'd Axis *upon which the generating Semi-circle turns, are call'd the* Poles *of the* great *Circle generated by this Circumvolution; the Fix'd Diameter its* Axis: *And they are the* Poles *and* Axis *of all the Parallel* Lesser Circles. Def. 3.

Scholium. IF we conceive any other *great* Circle of the *Sphere* generated after the same manner, we shall find its *Axis* at Right Angles to the Circle generated: And each of its *Poles* at a Quadrantal Distance, in the Arch of a *great*

Circle perpendicular to its Plane, from any Point assignable in its Circumference.

Def. 4. *If we conceive this* Axis *as representing that which the* Earth *turns round upon in* 24 *Hours, then the* great *Circle generated by the Circumvolution of the Semi-circle will represent the* Equator *in the* Terrestial Globe: *The Two Extremities of the* Axis *being its* Poles, *and those of all its* Parallels. *And the Plane of this* great *Circle continued every way till it meets with the* Fixed Stars, *will give us what we call the* Equator *in the* Heavens: *And the* Axis *continu'd thither will point out the two* Poles *of the* World.

Scholium 1. THE chief Parallels to the *Equator*, and the only ones usually drawn upon the *Globes*, are the Two *Tropical* and the two *Polar* Circles: The Two *Tropical* are those of *Cancer* and *Capricorn*; the former being 23° 29′ 00″ distant from the *Equator* towards the *North*; and the latter as much distant from the same towards the *South*: And if we conceive the Planes of these expanded every way till they cut the Surface of the *Sun*'s Apparent Annual Orb, the *Sun* will seem to us to move in the *Tropic* of *Cancer* in the Longest Day in *Summer*, and in that of *Capricorn* in the Shortest Day in *Winter*; from both which he returns again towards the *Equator*, which is the reason of their Names: And if we conceive as many Parallels to these Rightly drawn, 'twixt the Two *Tropical* Circles, as the Number of Days in a *Tropical* Year require, we shall have the Parallels which the *Sun* seems to move in from Day to Day all the Year round. The Two *Polar* Circles are distant the one from the *North*, and the other from the *South* Pole of the *World* 23° 29′ 00″.

Schol. 2. The *Equator* is so call'd, because when the

the *Sun* runs that Diurnal Circle the Days and Nights are every-where Equal; and, as it is a *great* Circle of the *Sphere*, it divides the *Earth* into Two Hemispheres, the *Northern* and the *Southern*: And as the *Earth* turns round upon its own *Axis* once in 24 Hours, the *Equator* compleats one Revolution in that Time; and therefore, as it is divided into 360 *Degrees*, it runs 15 *Degrees* in one *Hour*, 15 Minutes of a *Degree* in a Minute of *Time*, and 15 Seconds in one Second. Upon the *Equator* we reckon the Difference of the *Longitude* of Places in *Geography*; and from it towards the *North Pole*, or that *Pole* of the World which is visible to us, their *Northern*, and towards the *South* their *Southern Latitude*: And if we conceive the Plane of this great Circle expanded every Way till it meets with the *Fixed Stars*, we count their Differences of *Right Ascensions* upon it; and from it towards both the *Poles* of the *World* their *Declinations*: The *Northern* towards the *North*, and the *Southern* towards the *South Pole*.

If a great Circle *is perpendicular to the Plane of the* Equator, *the* Axis *of the* Equator *will be in its Plane; as it passes thro' the Centre of the* Sphere, *and stands at Right Angles to the Plane of the* Equator, *as well as the* Axis *it self does; and consequently it will pass through both the* Poles *of the* World: *And if it passes also thro' the* Vertex *of the Place we are in, it is call'd the* Meridian *of that Place. And a Number of Circles thus circumstantiated intersecting each other in the* Poles *of the* World, *and having its* Axis *for their common Section, are all* Meridians *to the Places upon the* Globe *thro' whose* Vertices *they pass. They are also call'd* Hour-Circles. *The Two most remarkable of these Circles describ'd upon the* Globe, *besides the proper* Meridian *of the Place, are that* Def. 5.

which passes thro' the First *Points of* Cancer *and* Capricorn, *and the Two* Poles *of the* World, *call'd the* Solstitial Colure; *and that which passes thro' those two* Poles *and the* First *Points of* Aries *and* Libra, *call'd the* Equinoctial Colure.

Scholium. THE *Meridian* divides the *Sphere* into its *Eastern* and *Western Hemispheres*; and upon the *Meridian* of the Place we are in we reckon our *Latitude*, and the *Declination* of the *Sun* and *Stars* from the *Equator*: Our *Latitude* being nothing else but the Arch of our *Meridian* intercepted 'twixt the Place we live in and the *Equator*; and the *Declination* of the *Sun* or any *Star*, an Arch of the same *Meridian* intercepted 'twixt the *Equator* and the *Sun*, or that *Star*: And the Difference of *Longitude* 'twixt any Two Places upon the Globe is nothing else but the Angle which their respective *Meridians* form at the *Pole* of the *World*, where all the *Meridians* intersect each other, which Angle is measur'd by an Arch of the *Equator* intercepted 'twixt those Two *Meridians*. When the *Sun* is upon the Upper part of our *Meridian* 'tis *Noon*, and from hence this *great* Circle of the *Sphere* has its Name; when on the Lower part 'tis *Midnight*. And whereas, by reason of the *Earth*'s Circumrotation upon the *Axis* of the *Equato* in 24 Hours, the *Sun* seems to move round the *Earth*, either in the *Equator* or a Parallel to it, in the same Time; if Circles are drawn upon the *Globe* at 15 *Degrees* Distance from each other in the *Equator*, and thro' both the *Poles* of the *World*, they are call'd *Hour-circles*; the *Sun* being an Hour in seeming to move from one of 'em to another; and the Intersections of their Planes with those of any other *great* Circles of the *Sphere* are what we call the *Hour-Lines* in *Dialling*, in those Planes. From one of the Points of the *Equator* where it is cut by the *Equinoctial*

Equinoctial Colure, *viz.* that which the *Sun* is in at the Celebration of the *Vernal Equinox* to us who have the *Northern Latitude*, we begin to count the Right Aſcenſion of the *Sun* and *Stars*: Which is nothing elſe but the Diſtance of that Point of the *Equator* with which they come upon our *Meridian* from that in which the *Sun* is at the time of our *Vernal Equinox*; or, from the *Firſt* Point of *Aries*. And upon the Planes of both theſe *Colures* the Reader will find the *Sphere* projected in the Sequel of this Book.

As the Earth *turns upon its own* Axis *in* 24 *Hours and thereby generates that* great *Circle which we call the* Equator; *ſo it is carried round the* Sun *in an Orbit 'twixt thoſe of* Mars *and* Venus *in the ſpace of a Year: And if we conceive a Right Line drawn from the Centre of the* Sun *thro' that of the* Earth *and continued till it falls in with the* Fixed Stars, *and imagine it to move along with the* Earth *'till it has finiſh'd one complete Periodical Revolution; it will mark out that* great *Circle of the* Sphere *amongſt the* Fixed Stars *which we call the* Ecliptic; *whoſe Plane is* Inclin'd *to the Plane of the* Equator 23° 29′ 00″, *and conſequently their* Axes *are ſo much* inclin'd *to each other: And their neareſt* Poles *are* 23° 29′ 00″ *diſtant.* Def. 6.

Scholium. THE Diviſion of the *Ecliptic* into Twelve Parts call'd *Signs*, with the *Aſtronomical* Characters by which thoſe *Signs* are repreſented, every body is acquainted with who has in the leaſt dipt into theſe Matters; ſo that I only hint at 'em in this Place. Upon the *Ecliptic* we count the *Longitude* of the *Fixed Stars* and *Planets*, beginning our Reckoning at that Point where the *Ecliptic* and the *Equator* interſect each other in the *Vernal Equinox*, call'd the *Firſt*

 Point

Point of *Aries*: And the Parallels to the *Ecliptic* are Parallels of *Celestial Latitude*. And if we imagin a Number of *great* Circles of the *Sphere* standing at Right Angles to the Plane of the *Ecliptic*, and consequently intersecting each other in its *Poles*; these are call'd Circles of *Celestial Longitude*: The Difference of *Celestial Longitude* 'twixt any Two *Phænomena* being nothing else, but the Angle which their respective Circles of *Longitude* form at either *Pole* of the *Ecliptic*, which Angle is measur'd by an Arch of the *Ecliptic*, intercepted 'twixt those Two Circles. And the Arch of any one of these Circles intercepted 'twixt the *Ecliptic* and any *Star* or *Planet*, is the *Latitude* of that *Star* or *Planet*. The Two *Tropicks*, which I have already taken notice of as Parallel to the *Equator*, are form'd by the seeming Diurnal Motions of the First Points of *Cancer* and *Capricorn*, whence they have their Names. And the *Two Polar* Circles derive theirs from the seeming Diurnal Circumvolution of the *Poles* of the *Ecliptic* round those of the *World*; from whence they deduce their Origin. The *Ecliptic* has its Name from hence, that all the *Eclipses* of the *Sun* and *Moon*, &c. are perform'd either actually in, or very near the Circumference of that *great* Circle.

Def. 7. *If we conceive a* great *Circle of the* Sphere *cuting the* Meridian *of our Place at Right Angles, distant from our* Zenith *a Quadrant or* 90° 00″ *this is what we call the* Horizon *of the Place.*

Scholium 1. THE *Horizon* divides the Visible from the Invisible, or the Upper from the Lower *Hemisphere* of the *Globe*: And when the *Sun*, or any *Star* or *Planet* appears in it, it is said to *Rise*; when they begin to disappear, or sink under it, they are said to *Set*. When

the *Horizon* is consider'd as a *great* Circle of the *Sphere*, it is what we call the *Rational Horizon*: But the *Geographers* sometimes call so much of the *Earth*'s Surface as is taken into the Eye, the Eye being turn'd round to all the Points of the Compass, with the Circular Termination of that Vision, by the Name of the *Horizon*, (from the *Greek* of which Circle terminating our Vision it has its Name, and from this we borrow the Name of that *great* Circle of the *Sphere* which we call the *Rational Horizon*) which is of no farther Consideration in this Place, after I have observ'd that, for Distinction-sake, they call it the *Sensible Horizon*. Upon the *Horizon* we count all the *Points* of the *Compass*, or *Bearings* of Places, in *Geography*; and to it we refer the *Azimuths* of the *Sun* and *Stars*; beginning either at the *North* or *South* Points of our *Meridian*, or at those Points in the *Horizon* where it is intersected by a *great* Circle which stands at Right Angles to our *Meridian*, and passes thro' our *Vertex* at the same time, which is call'd the *Prime Vertical*: those Points of Intersection being the due *East* and *West* Points. And if we conceive more *great* Circles at Right Angles to the Plane of the *Horizon*, cutting each other in the *Vertex* and the opposite *Pole* of the *Horizon*, at $11_{0}\ 15'\ 00''$ distance from each other; the Points of their Intersection with the *Horizon* will be the Points of the *Compass*: And whatever Place upon the *Earth* is found in a Right Line drawn from the Place we are in, which is conceiv'd as the Centre of our *Horizon*, to any of these Points of Intersection, is said to be upon that *Point* of the *Compass*, with respect to the Place from which, as a Centre, that Line is drawn. Whatever *Celestial Phænomenon* is found upon any one of these Circles, different from the *Meridian* of the Place and the *Prime Vertical*,

tical, is ſaid to be upon ſuch an *Azimuth* from the *North* or *South*, or to have ſuch an *Amplitude* from the *Eaſt* or *Weſt*: If it is upon the *Eaſtern* ſide of the *Meridian*, and upon the *Prime Vertical*, it is due *Eaſt*: If it is on the *Weſtern* ſide of the *Meridian*, and upon the ſame *Vertical* Circle, it is due *Weſt*, &c. If as I have already obſerv'd, a *Star* in the *Heavens*, or any Place upon the *Terreſtial* Globe, is betwixt the *Equator* and the *North Pole* of the *World*; then the former is ſaid to have *Northern Declination*, and the latter *North Latitude*; And if either of 'em is betwixt the *Equator* and the *South Pole* of the *World*, then their *Latitude* or *Declination* are *South*. If a *Star* is 'twixt the *Ecliptic* and its *North Pole*, its *Latitude* is *North*; if the contrary, it has *South Latitude*. And the Arch of a *Vertical* Circle intercepted 'twixt that Point of it which any *Phænomenon* is in, and the *Horizon*, is call'd the *Altitude* of that *Phænomenon* above the *Horizon*; the Complement of that Arch to 90 *Degrees* being its Diſtance from the *Vertex*. And if a Parallel to the Plane of the *Horizon* paſſes thro' that Point where the *Phænomenon* is found, it is call'd the *Parallel* of its *Altitude*.

Schol. 2. Theſe are the Chief Circles of the *Sphere* which the Reader will be taught to Project *in Plano* in the Sequel of this Diſcourſe. The *Equator*, with its Parallels; and thoſe *great* Circles which cut its Plane at Right Angles, and each other in the *Poles* of the *World*; call'd in the *Terreſtial Globe* Circles of *Longitude*, or *Meridians* of the Places they paſs over; and in the *Celeſtial* Circles of *Right Aſcenſion*: The *Ecliptic*, with its Parallels; and thoſe *great* Circles which cut its Plane at Right Angles, and each other in its *Poles*; which are call'd Circles of *Celeſtial Longitude*.

gitude. And the *Horizon*, with its *Parallels*; and those *great* Circles of the *Sphere* which stand at Right Angles to its Plane, and intersect each other in its *Poles*, the *Vertex* or *Zenith*, and the *Nadir*; which set out the *Points* of the *Compass* upon the *Horizon*, and upon which we measure the *Altitude* of any *Celestial Phænomenon*, or its Distance from the *Vertex*, are called *Vertical* or *Azimuth* Circles. I have just hinted at some of their general Uses; the Reader, I expect, will scarce come wholly unacquainted with the Uses of them to the Perusal of this Piece: Or, if he does, he may meet with 'em at Large in any Common *Elementary* Introduction to the *Uses* of the *Globe*; or in the following Chapters of this Book, where I apply the *Circles* of the *Sphere* Projected *in Plano* to the Solution of *Astronomical*, *Geographical*, Problems, *&c.* I shall only take notice farther here, that the Planes of the *Equator* and the *Ecliptic* are always *Inclin'd* to each other 23° 29′ 00″. But that the Planes of the *Equator* and the *Horizon* are in some Places *Parallel* to each other, as to those who live directly under either the *North* or *South Poles* of the *World*: In some Places *Perpendicular*, or at Right Angles to each other; *viz.* to all such as live under the *Equator*: And in all other Places they are *inclin'd*, or *Oblique* to each other; and that the more, the less the Latitude of the Place we are in, and the less the greater our Latitude. For which reason the *Sphere*, upon account of its Position, or, more properly, of the Situation of the Place we are in, may be consider'd either as *Right*, *Parallel*, or *Oblique*.

A Right *Sphere is that in which the Planes of the* Horizon *and the* Equator *stand at* Right *Angles to each other; as they do to all those who live precisely under the* Equator. *A* Parallel Sphere *is that in which the Planes of the* Horizon *and the* Def. 8.

the Equator *are* Parallel *to each other*, *or more properly*, *coincide one with another*; *as they do only to those who live exactly under the* North *and* South Poles *of the* World. *And an* Oblique Sphere *is that in which these Two Planes are* inclin'd *to each other*: *Which they are to all the Inhabitants of the* Earth *which have either* North *or* South Latitude, *and live not precisely under either of the* Poles.

Scholium. THE different Lengths of the Days and Nights, Risings and Settings of the *Stars*, &c. in these different Positions of the *Sphere*, the Reader must be acquainted with, who has dipt into the very *First* Rudiments of *Geography* and *Astronomy*; and I shall have occasion to take notice of some of 'em in the Sequel. I shall only take notice farther here, That the Inhabitants of the *Earth* living in different Parts of the Surface of the *Globe*, are call'd by different Names by the *Geographers*, accordingly as their Habitations are situated with respect to one another, or to the *Sun*: Those being call'd *Antæci*, who live under the same *Meridian*, but in Parallels of Latitude one as much *North* as the other is *South*: Those *Periæci*, who live in the same Parallels, and in opposite Points of the same *Meridian*: And those *Antipodes* who live in opposite Parallels, and in opposite Points of the same *Meridian*. The *Amphiscii*, are those who live between the Two *Tropics*: the *Periscii*, those who live between the Two *Polar* Circles and their respective *Poles*: And the *Heteroscii*, those who live between the *Tropics* and the *Polar* Circles. The Reasons of these Names, and the *Phænomena* which happen to those who live in these Places, or with these respects to each other, every one may see in the Writers upon the *Uses* of the *Globes*.

Projection

Projection in General, is, the Art of representing any Points, Lines, Surfaces, *or* Solids, *upon any assignable* Planes; *as they appear, or are mark'd out upon those* Planes, *by Lines drawn from them, or any, or all the Parts of them, to the* Eye, *intersecting, or, if need be, continu'd to, the* Plane *they are to be* Represented *upon.* Def. 9.

The Projection *of the* Sphere, *in Particular, is the Art of Drawing its Circles upon a* Plane, *so as they shall be mark'd out upon it by Right Lines passing from the* Eye *to every Point in their Circumferences, and either intersecting or continu'd to the* Plane *of* Projection. Def. 10.

Scholium. THE *Rays* which are drawn from every Point of the Circumference of any Circle of the *Sphere* to the *Eye*, form a *Cone*; of which the Point where they are collected in the *Eye*, is the *Vertex*; and that Circle itself the *Base*: And if we imagin a *Plane* interpos'd 'twixt the *Eye* and that Circle cutting the *Cone*, the Circumference of the Section of the *Cone* made by that *Plane* will be the representative of that Circle in the *Plane*: if we imagin the *Plane* to stand beyond the Circle to be projected upon it, then the *Cone* of *Rays* continu'd till it cuts the *Plane* will give us a Section of that *Plane* whose Circumference shall represent the Circle to be projected: And these Representatives will be different, accordingly as the *Planes* of Projection shall be *Perpendicular* or *inclin'd* to the *Axis* of the *Eye* continu'd. If the *Plane* of the Projection is that of a *great* Circle of the *Sphere*, having a Line drawn from the *Eye* to its Centre Perpendicular to that *Plane*; then all its Parallels 'twixt the *Eye* and the *Plane* of Projection, are Projected by continuing the *Cones* of *Rays* from the *Eye* to their Circumferences 'till they

they meet with the *Plane*; and all its Parallels beyond the *Plane* are mark'd out by the Sections made by the *Plane* in their respective *Cones* of *Rays*: And all the Inclin'd *great* Circles of the *Sphere*, being half above and half below the *Plane* of Projection, the former halfs are mark'd out as the Parellels betwixt the *Plane* and the *Eye*; and the latter, as those beyond the *Plane*; and their Parallels in like manner, accordingly as they are in whole or in part above or below the *Plane* of Projection, that is, nearer or farther off than it is from the *Eye*: and so for the rest. And whereas we may conceive the *Eye*, either as in the *Surface* of the *Sphere*, in the *Pole* of the *great* Circle, upon the *Plane* of which the other Circles, whether *great* or *small*, are to be Projected; Or, at an *infinite* Distance from it; Or, in some proper intermediate Point 'twixt these two extremes, projecting the rest of the Circles of the *Sphere* upon the *Plane* of any one of its *great* Circles: Hence arise Three different Ways of Projecting the *Sphere in Plano*: The *first* of which is call'd *Stereographic*; the *second*, *Orthographic*; and the *third*, when the *Eye* is at a due Distance for distinct Vision, is call'd *Scenographic* Projection, and coincides with what we commonly call *Perspective*. The Last of these *Three* kinds of Projections I shall have nothing to do with in this Place. But the *Two* former will take up the *Two First* Chapters in the Work before us.

Def. 11. *The* Orthographic Projection *of the* Sphere, *is the Art of Drawing all the rest of its Circles upon the* Plane *of any one of its* great *ones, so as they shall appear upon it to the* Eye *at an* infinite *Distance; having its* Axis *continu'd* perpendicular *to the* Plane *of that great Circle upon which the Projection is made: and passing thro' its Centre.*

The

The Stereographic Projection *of the* Sphere, *is the* Def. 12.
Art of Drawing all the rest of its Circles upon the Plane *of any one of its* great *ones, so as they shall appear upon it to the* Eye *in the* Surface *of the* Sphere *in one of the* Poles *of that* great *Circle: In which Case the* Axis *of the* Eye *continu'd, will be* perpendicular *to the* Plane *of that* great *Circle upon which the Projection is made, and pass thro' its Center.*

Scholium. IN order to deliver the Art of Projecting the *Sphere*, both *Orthographically* and *Stereographically*, in as easy and perspicuous a manner as I can think of; I shall allow each of 'em their distinct Chapter, and in each Chapter observe this Method: *Viz.* First, *I will briefly and perspicuously demonstrate under what* Figure *all the rest of the Circles of the* Sphere, *whether* great *or* small, *appear upon the* Plane *of that Circle upon which the* Projection *is to be made.* I. Secondly, *How we may determine the Requisites for the Delineation of the rest of the Circles of the* Sphere *upon the* Plane *of the* Projection. II. Thirdly, *I will deduce the Art of* Measuring Projected *Circles, whether* great *or* small; *or of cutting off Arches of 'em, representing any assignable Arches of those Circles of the* Sphere *from which they are deriv'd.* III. And Fourthly, *I will exemplify the Rules discover'd under these* Three *Heads, in the actual* Projection *of the* Sphere *upon the* Planes *of one or more of its* great *Circles; and in the* Mensuration *of the Representatives of the Circles of the* Sphere *in the* Plane *of the* Projection. IV. After which, the *Series* of my Design will lead me to shew the *Use* of the *Projection* of the *Sphere in Plano*, in the Solution of *Astronomical, Geographical* Problems, *&c.* in the succeeding Chapters of this Book.

Axiom.

Axiom.

The *Place* of any *Visible Point* upon the *Plane* of the *Projection* is there in that *Plane*, where a Right Line drawn from the *Eye* to that Point intersects the *Plane*: Or, if the *Plane* is beyond the *Visible Point*, where that Right Line continu'd meets with the *Plane* of the *Projection*.

Scholium. THIS *Axiom* is the great Fundamental which all manner of *Projections* are built upon: And from which all the Rules for the *Projection* of the *Sphere in Plano* are primarily deduc'd; and into which they are ultimately resolv'd. And it holds not only of *Points*; but of all *Lines*, as they consist of *Points*; and of all *Surfaces*, as they may be conceiv'd as consisting of *Lines* which consist of Points: And of *Solids* too, as their Representatives in the *Plane* of the Projection are no other than *Plane Surfaces* of some *Figure* or other, which may be resolv'd into *Lines*, and those into *Points*: As every one must see who understands in the least what *Projection* is, without any farther Illustration. I need not advertise the Reader here, that the *Plane* of that *great* Circle of the *Sphere*, upon which the other Circles are to be projected, is conceiv'd as something *transparent*, in order that the *Rays* from the *Eye* may pass it, and mark out upon it the Places of those Circles or Parts of Circles which are, with respect to the *Eye*, beyond the *Plane*: And that the *Sphere* is suppos'd to be divested of all *Matter* but that of which its *Circles* consist, that the *Places* of both those Circles and Parts of Circles which are beyond the *Plane*, and those 'twixt it and the *Eye*, may be view'd upon the *Plane* of the *Projection*.

CHAP.

CHAP. I.

Of the ORTHOGRAPHIC Projection of the SPHERE.

SECT. I.

Of the Appearance of the Circle of the Sphere *upon the Plane of the Projection.*

LEMMA I.

In Orthographic *Projections, all Right Lines drawn from any Points of a Circle of the* Sphere *thro' the Plane upon which that Circle is to be projected, to the Eye at an infinite Distance, are Parallel amongst themselves.*

DEMONSTRATION.

LET the Right Lines *d b* and *e b* be drawn from the Extreme Points *d* and *e* of the Diameter *d e* of the Circle *d h e i*, intersecting the Plane of the Projection in *p* and *n*: I say, If the Point *b* be conceived at an infinite Distance from the Point *m* in the Plane of the Projection, the Lines *b d* and *b e* will coincide with the Right Lines *d g* and *e f*, which are drawn Parallel to each other from the Points *d* and *e*. Conceive the Point *b* removed to *a*, then the Lines *b d* and *b e* will become *a d* and *a e*, but the Angles *d a c*, and *c a e*, which are respectively equal to *a d g* and *a e f*, by the 29 of the *Fig.* 1.

the 1 of Euclid, are leſs than the Angles *d b c* and *c b e*, by the 16 of the 1 of Euclid, which are reſpectively equal to *b d g* and *b e f*; conſequently the farther the Point *a* is remov'd from *b*, or, which is the ſame, the farther the Point *b* is removed from *c*, the leſs will be the Angle *d b c*, by the 21 of the 1 of Euclid; and the nearer will the Lines *b d* and *b e* approach to the Lines *g d* and *e f*, till it arrives at an infinite Diſtance, when the Angle *d b e* will vaniſh. The Lines *d b* and *b e* will coincide with the Lines *g d* and *f e* and become Parallel to each other.

Q. E. D.

Scholium. This Paralleliſm of all Lines drawn from any Points of the Circumference of a Circle projected *Orthographically* upon an aſſign'd Plane, is a peculiar Characteriſtic of this kind of Projections; reſulting from the ſuppos'd infinite Diſtance of the Eye from the Plane of the Projection: And 'tis manifeſt, that it holds in Lines drawn from any other Points whatever, whether in Lines, Surfaces, or Solids; as the Points *d* and *e* may be conceiv'd in any other Figure as well as a Circle, in a Right or any kind of Curve Line, in a Surface or a Solid. It muſt only be noted here, that by *infinite Diſtance*, in this Place, is not meant ſtrictly ſuch, but a Diſtance ſo very large as in which the Angles *a e f* and *a d g* ſhall be leſs than any Angle aſſignable, or in a ſtate approximating infinitely near to Evaneſcency: So that Right Lines from the Object thro' the Plane of Projection to the Eye are not ſtrictly Parallel, but approach infinitely near to a Paralleliſm betwixt themſelves; ſo as to make theſe Projections, to all Intents and Purpoſes, the ſame as if they were actually abſolutely Parallel. 'Tis *Infinity* in a *Phyſical* Senſe, with reſpect to our Organs of Viſion, when

when the Evanescent Angles are indiscernible, and consequently the Right Lines from each Point of the Object to be projected thro' the Plane of Projection, continued to the Eye, may be considered as *Physically* Parallel betwixt themselves. This I thought fit to Note in this Place; because it takes place not only in the preceding *Lemma*, but in all the *Demonstrations* relating to *Orthographic* Projections.

LEMMA II.

In a Circle, the Rectangle under the Segments of the Diameter is Equal to the Square of the Ordinate.

DEMONSTRATION.

LET the Ordinate *a b* be perpendicular to the Diameter *c d*, and draw the Lines *a c* and *a d*. The Angle *c a d* being in a Semi-circle is Right; by 31. III. *Euclid*: and the Triangles *a b c*, *a b d*. and *c a d*, are Similar; by 8. VI. *Euclid*: Therefore *c b* : *a b* : : *ab* : *b d*. And, by 17. VI. *Euclid*, the Rectangle under *c b d* is Equal to the Square of *a b*. *Fig.* 2.

Q. E. D.

LEMMA III.

In an Ellipsis, *the Rectangles under the Segments of the Transverse Axis, are as the Squares of the Ordinates by which those Segments are made.*

DEMONSTRATION.

IN the Cone *a b c l g*, let the Circles *t s r q* and *n o e p* be parallel to the Circular Base *c l a g*; and let *d n t f r e* be an *Ellipsis*, having its Ordi- *Fig.* 3.

nates *me* and *hr* perpendicular to the Diameters of the Circles *op* and *sq*, as well as to its own Tranſverſe Axis *dmhf*. I ſay, The Rectangle *dmf* under the Segments *dm*, *mf* of the Tranſverſe Axis *dmf* is to the Square of *me*, as the Rectangle *dhf* under the Segments *dh*, *hf* is to the Square of *hr*.

For, the Ratio of the Rectangle *dmf* to that of *dhf* is compounded of the Ratio of *dm* to *dh*, and *mf* to *hf*; by 23. VI. *Euclid.* But as *dm* is to *dh*, ſo is *mp* to *hq*, by the Similar Triangles *dmp* and *dhq*; and as *mf* is to *hf*, ſo is *mo* to *hs*, by the Similar Triangles *fhs* and *fmo*: Therefore the Ratio of the Rectangle *dmf* to the Rectangle *dhf* is compounded of the Ratio of *mp* to *hq*, and *mo* to *hs*. But the Rectangle *pmo* is to the Rectangle *qhs* as *mp* to *hq*, and *mo* to *hs*: Therefore the Rectangle *dmf* is to the Rectangle *dhf*, as the Rectangle *pmo* is to the Rectangle *qhs*. But in the Circles *onpe*, *stqr*, the Rectangle under *om p* is equal to the Square of *me*, and that under *qhs* is equal to the Square of *hr*, by the precedent *Lemma*: Therefore the Rectangle under *dmf*, is to the Rectangle under *dhf*, as the Square of *me* is to the Square of *hr*.

Q. E. D.

Corollary 1. The Ordinates of a Circle, whoſe Diameter is the Longer Axis of an Ellipſis, are to the Ordinates of the contain'd Ellipſis, as the Tranſverſe Axis of the Ellipſis is to its Conjugate Axis. For the Rectangle *dca* is to the Rectangle *dba*, as the Square of *ce* is to the Square of *bf*; by *Lemma* II. And the Rectangle *dca* is to the Fig. 4. Rectangle *dba*, as the Square of *ch* is to the Square of *bg*; by *Lemma* III. Therefore the Square of *ec* is to the Square of *fb*, as the Square of *hc* is to the Square of *bg*. Therefore (by

22.

22. VI. *Euclid.*) $ec : fb :: hc : gb$. Or, $ec : hc :: fb : gb$. And twice $ec = ad$, Transverse Axis: twice hc Conjugate Axis :: $fb : gb$.

Q. E. D.

Coroll. 2. Hence we are furnish'd with a Geometrical Method of describing an *Ellipsis*, having its Longer and Shorter Axis: For bissecting its longer Axis ad in the Point c, and drawing the Shorter Axis at Right Angles to it thro' that Point; and circumscribing a Circle about the Longer Axis, whose Centre is c: If we let fall any Number of Ordinates at pleasure from any Points in the Circumference of the circumscrib'd Circle at Right Angles to the Longer Axis $abcd$, we may find the corresponding Ordinates in the Ellipsis. As if fb is let fall in the Circle, 'tis $ec : hc :: fb : bg$, which will give the Point g, thro' which the Ellipsis is to pass. Thus if we lay off ec, (*Fig.* 4.) and make ec equal to it, (*Fig.* 5) and eb to fb; and at Right Angles to ec, at the Point c erect the Perpendicular $ch = ch$, *Fig.* 4. and draw the Line eh; and at the Point b of the Line $eb = bf$ erect the Perpendicular bg, that will be equal to bg in *Fig.* 4. For $ec : ch :: eb = fb : bg$; by 12. VI. *Euclid.* Therefore bg (*Fig.* 5.) set off in the Circular Ordinate bf from b to g, will give g a Point thro' which the Ellipsis must pass: And so for the rest. There are a great many other Ways of drawing Ellipses to be met with in the *Conic* Writers: I have mention'd this here, because it flows from a Property of the Ellipsis, which I shall shortly have occasion to make use of. And for all other, the Reader must be refer'd to those who treat purposely on the *Conic Sections*. *Fig.* 5.

PROP. I. Theorem.

If a Circle is Perpendicular to the Plane of the Projection, having its Plane coinciding with, or Parallel to, Lines drawn from the Eye at an infinite Distance; it is projected into a Right Line Equal to its Diameter.

DEMONSTRATION.

Fig. 6. 1. LET the Circle *a b d u* be projected upon the Plane *g p q f*, to which it is perpendicular, by the Eye, at an infinite Distance, having its Axis continued *i e b c u* coinciding with the Plane of the Circle to be projected; and therein perpendicular to the same Plane: As the Axis of the Eye continued, and the Circle to be projected, are in the same Plane, *i e c u* must be a Right Line by 1. XI. *Euclid*; and consequently will project the Points *b* and *u* of the Circle in the Point *e* of the Plane, in the common Section of the Planes *a d f g* and *g p q f*: As it must be in both the Planes. If *i e c* changes its Place so as to become successively *h g a* and *k f d*, as it still continues in the same Plane, it will project *a* in *g* and *d* in *f* in the same Plane with *e*, and therefore in the common Section of the Planes *a d f g* and *g p q f*: And the Case is the same with all the intermediate Parts of the Circle *b a u d*, which will be represented by a Series of Points in the Intersection of the Two Planes 'twixt *g* and *f*; Therefore *g f*, which represents the Whole, must be a Right Line, by 3. XI. *Euclid*, equal to *a d*; because *a g* and *d f*, *a d* and *g f*, are Parallels; by 34. I. *Euclid*.

2. If the Lesser Circle *o m n x* parallel to *a b d u* is to be projected in the same Plane, it will be also repre-

represented by the Right Line *p q* equal to its Diameter *o n*: For if we suppose Circles cutting these Two in every Point of their Circumference, and standing at Right Angles to 'em, Parallel to *b c u*; 'tis plain, that Lines drawn from any Points of 'em, as from *b* and *m* in one, *u* and *δ* in another, and *o* and *μ* in a third, to the Eye at an infinite distance, will be Parallel to each other; by *Lemma* I. All the Lines therefore of which the Plane *o p q n* consists, will be Parallel to those of *a g f d*: Therefore *p t q* and *g e f* will be parallel; by 16. XI. *Euclid.* Therefore *p t q* is a Right Line, which is apparently the Representative of *m o x n*, and Equal to *o c n*; by 34. I. *Euclid.*

Q. E. D.

Scholium 1. The Reader will perceive in the Sequel, that in *Stereographic* Projections, Lesser Circles perpendicular to the Plane of the Projection are projected in Circles; and in the next Proposition save one, that in *Orthographic* Projections, Circles inclin'd to the Plane of the Projection, are projected in *Ellipses*; and so are, in strict Geometry, Perpendicular Lesser Circles too; as every one may prove to himself, who rightly understands the *Scholium* to the *First Lemma*. But as the Eye is suppos'd, in these Cases, at an indeterminate great Distance from the Plane of the Projection; compar'd with which, the Semi-diameter of that Plane may be look'd upon as nothing; and consequently as its Axis continued passing thro' the Centre of that Plane, may also be considered as coinciding with the Plane of each Perpendicular Circle: Hence it must follow, that these *Ellipses* will have their *Foci* infinitely near the very Ends of their *Transverse* Axes; and consequently their *Conjugate* Axes will vanish; and they will appear, to all Intents and Purposes, Right Lines.

Schol. 2. If the Plane of the Projection be a Great Circle of the *Sphere*, one of whose Diameters is *a c d*, and which stands at Right Angles to *a b d u*, and its Parallels; *a θ x n* being part of its Plane: 'Tis evident that *θ x* = *p q* will represent *o m n x*; and *a d* = *g f* will represent *a b d u*.

PROP. II. Theorem.

If a Circle is Parallel to the Plane of the Projection, and Perpendicular to the Axis of the Eye continued passing thro' its Centre to the Centre of that Plane, it is projected into a Circle Equal to itself.

Demonstration.

THis is plain: For all Lines drawn from the Circumference of this Circle to the Eye at an infinite Distance; and passing through, or, if need be, continued to the Plane of the Projection, will be parallel to each other, and to the Line passing thro' the Centres of the Circle to be projected, and the Plane of the Projection; by *Lemma* I. Therefore if we conceive 'em drawn from every Point of the Circumference of the Circle to be projected, and continued to the Plane of the Projection, as they are at Right Angles to that Plane, and to that of the Circle to be projected, they will evidently form a Right Cylinder; by the Nature of which the Base in the Plane of the Projection is a Circle, equal to the Circle projected.

Q. E. D.

PROP

PROP. III. Theorem.

If a Circle is inclin'd to the Plane of the Projection, to which Plane Lines drawn from the Eye at an infinite Distance are at Right Angles, the Circle is projected into an Ellipsis; *having its Longer Axis equal to its Diameter, and its Shorter to twice the Co-sine of its Inclination to the Plane of the Projection, half its Longer Axis being the* Radius.

Demonstration.

LET the Circle *a d b e* be inclin'd to the Plane of the Projection *d g e f*, having one of its Diameters *e c d* common to itself and the Plane of the Projection; to which *the principal Ray*, or a Line drawn from the Eye at an infinite Distance thro' the Centre of the Plane at Right Angles with it, is Perpendicular; but to all the rest, as not to be found in the same Plane, Oblique: Let fall Perpendiculars from *a* to *g*, from *l* to *k*, and so from how many Points soever you please of the inclin'd Circle to the Plane of the Projection: these Perpendiculars will be Parallel among themselves, and to *the principal Ray c p q* continued; and will project the Circle upon the Plane into an Ellipsis *d g k e f*. Fig. 7.

For the Perpendiculars *a g* and *k l* are Parallel, by *Lemma* I. Draw therefore *l h* parallel to *a c*, and *k h* parallel to *g c*; and suppose the Diameter *a c b* at Right Angles to the Diameter *e c d*. The Square of *a c* is equal to the Rectangle under *e c d*, and the Square of *l h* is equal to the Rectangle under *e h d*; by *Lemma* II. Therefore □ *a c* : □ *l h* : : ▭ *d c e* : ▭ *d h e*. And in the Triangles *a c g* and *l h k* all the Sides being respectively

ſpectively Parallel, and the Angles at g and k Right; and $acg = lhk$, as form'd by the ſame Inclination of the ſame Planes; therefore they are Similar; and $ac : gc :: lh : kh$. Therefore (by 22. VI. *Euclid*) $\square\, ac : \square\, gc :: \square\, lh : hk$. Therefore $\boxed{}\, dce \,(= \square\, ac) : \square\, gc :: \boxed{}\, dhe \,(= \square\, lh) : \square\, kh$. Not in a Ratio of Equality, as in the Circle; becauſe the Square of gc is apparently leſs than the Square of ac, and the Square of kh than the Square of lh. Therefore (by *Lemma* III.) the Points g and k are in an *Ellipſis*. And ſo for the reſt. And the Longer Axis is viſibly Equal to the Diameter of the Circle ecd: And the Shorter to twice gc, which is the Co-ſine of acg to the *Radius* cr or ac the Inclination of the Circle to be projected to the Plane of the Projection.

Q. E. D.

Schol. 'Tis plain, that Circles can only be either *Perpendicular*, *Parallel*, or *Inclin'd* to the Plane of the Projection; and conſequently, the three foregoing Propoſitions demonſtrate under what Figures all Circles that can be drawn upon it will appear; *viz.* Either as *Right Lines*, *Circles* or *Ellipſes*, in *Orthographic Projections*.

SECT.

SECT. II.

Of the Methods of Determining the Requisites for the Delineation of the rest of the Circles of the Sphere *upon the Plane of the Projection.* Sect. II.

PROP. I.

If a Circle is Perpendicular *to the Plane of the Projection, it is projected by drawing a Right Line from one Point to the other, where it intersects that Plane.*

DEMONSTRATION.

IT is projected into a Right Line equal to its Diameter by *Prop.* I. *Sect.* I. And that Diameter being in the Plane of the Projection, reaches from one Point of Intersection to the other.

Q. E. D.

Coroll. Hence it is easy to discover what Perpendicular Circle is represented by any projected Right Line; *viz.* If we know in what Points that Line intersects the Plane of the Projection.

PROP. II.

If a Circle is Parallel to the Plane of the Projection, its Centre is the same with that of the Plane of the Projection, and its Radius *is Equal to the Co-sine of its Elevation above, or Distance from that Plane, to the* Radius *of the Projection.*

DEMONSTRATION.

A Circle Parallel to the Plane of the Projection is projected into a Circle Equal to itself; by

by *Prop.* II. *Sect.* I. Therefore the *Radii* of the Primitive and Representative must be Equal. But by the Nature of the *Sphere*, the *Radius* of any Parallel to a Great Circle is the Co-sine of its Distance from that Circle to which it is Parallel. Therefore the *Radius* of its Representative is the same.

Q. E. D.

Coroll. Hence we may discover what Circle is represented by any projected Circle in *Orthographic Projections*; *viz.* By taking its *Radius*, and finding what it is the Sine of upon the *Sector* open to the *Radius* of the Projection. For so much is the Primitive Circle distant from one of the Poles of the Projection: Or taking that from 90 Degrees, the Residue is the Distance of that Parallel from its Great Circle. Only here the Reader must observe, that all Lesser Circles parallel to the Plane of the Projection, which are equidistant from each Pole of that Plane, are represented by the same Circle in the Projection; so that each Circle within that Plane represents Two Lesser Circles of the *Sphere*, one at a given Distance from the Plane towards the Eye, and the other at the same Distance from the Plane towards that Pole which is opposite to that which is nearest to the Eye.

PROP. III.

If a Circle is Inclin'd *to the Plane of the Projection, the* Ellipsis *which represents it is projected by laying of its Longer Axis, which is always Equal to its Diameter, from one Point to the other, in that Line where the Plane passing thro' the Diameter of the Circle to be projected intersects the Plane of the Projection: And, bissecting that Axis, the Shorter is determin'd by*

by setting off from that Point at Right Angles to the Longer Axis on each side the Co-sine of its Inclination *to the Plane of the Projection, to half the Longer Axis as the* Radius.

DEMONSTRATION.

THis is a manifest *Corollary* from *Prop*. III. *Sect.* I. And having the Longer and Shorter Axis, the Representative *Ellipsis* may easily be drawn by *Corol*. 2. *Lemma* II. *Sect*. I.

Q. E. D.

Coroll. Hence we may easily find what Circle is represented by any *Ellipsis* in *Orthographic Projections*. For if it is a *Great* Circle, its *Semi-conjugate* Axis is the Co-sine of its Inclination to the Plane of the Projection: If it is a *Lesser* Circle, its Inclination is discover'd the same way; and its Distance from its Parallel *Great* Circle, by the Length of its *Semi transverse* Axis; which is equal to the Co-sine of its Distance from, or Elevation above, the *Great* Circle, to which it is Parallel, to the *Radius* of the Projection. Only here also the Reader may observe, that the *Ellipsis* which represents any *Inclin'd* lesser Circle, represents also another Lesser Circle parallel to it, and equidistant with it from that Great Circle of the *Sphere* to which they both are parallel, on the contrary Side of that *Great* Circle: As every one must see who understands *Prop*. III. *Sect*. I.

SECT.

SECT. III.

Sect. III. *Of the Mensuration of Circles projected* Orthographically, *whether* Great *or* Small.

PROP. I. Problem.

If a Circle is projected into a Right Line, *to cut that Line into Parts Representative of any assign'd Divisions of that Circle.*

SOLUTION.

Fig. 8. DIvide the Right Line propos'd as *a b* into Two equal Parts in the Point *k*; and upon *k* as a Centre describe the Circle *a d b g*, which will be equal to the Circle projected by *Prop.* I. *Sect.* I. And if it is suppos'd divided into Six Parts in the Points *a*, *e*, *d*, *b*, *f*, and *g*; the opposite Parts *e a*, *a g*, &c. being equal: Lines drawn from *e* to *g*, and *d* to *f*, will cut off *h a* answering to *e a* or *g a*; *i h* answering to *d e* or *f g*; and *b i* answering to *d b* or *b f*. Because (by *Lemma* I. *Sect.* I.) Lines drawn from any Points in a Circle, as *a*, and *g*, through the Plane of the Projection, as *a l*, to the Eye at an infinite Distance, will be parallel to each other: And consequently the Parallels *g e* and *a l* will cut off *h a* equal in Representation to *a e* or *a g*.

Q. E. F.

Schol. 1. The Representative of an Arch of a Circle *Orthographically* projected in a Right Line, is equal to the Versed Sine of that Arch to the *Radius* of the Circle to be projected. For *h a* the Representative of the Arch *e a* or *g a* is the Versed Sine of either of those Arches, or the Angle *e k a* or *a k g*, to the *Radius k a*. Or the Co-sine of *e a* taken off from *k* to *h*, leaves *h a* the Representative of *e a*.

Schol.

Schol. 2. If a Right Line which is the Projection of a Circle is cut into an assigned Number of Parts, we may know what Arches of the Circle each Division represents by the Reverse of the Process of this Problem: *Viz.* By first drawing the Circle about it, as in the Problem, and then drawing Perpendiculars through the Points of Division, and continuing them to the Circumference of the Circle; for they will manifestly cut the Circle so as its Representative Right Line is cut.

PROP. II. Problem.

If a Circle is projected into a Circle, to cut the Projected Circle accordingly as that which it represents is cut.

Solution.

THE Projected Circle is equal and like to the Primitive it represents in all respects; by *Theor.* II. *Sect.* I. Therefore equal Arches of it answer to equal Arches of the Primitive: Which may easily be taken, as the Division of the Primitive requires.

Q. E. F.

PROP. III. Problem.

If a Circle is projected into an Ellipsis; *to divide the* Ellipsis *into any Number of Parts, answering to any assign'd Divisions of the Circle so projected.*

Solution.

LET the Circle *a d b e* be projected into the *Ellipsis g d f e*, and let the Quadrant of the *Ellipsis g e* be requir'd to be divided so as the Quadrant of the Circle *a e* is divided in *k*. Draw a Circle upon the Plane of the Projection *d h e i* equal to *a e b d*, and divide its Quadrant *h e* in *m* *Fig.* 9.

I as

as *a e* is divided in *k*: And from *m* let fall a Perpendicular *m p* upon *d e* the Transverse Axis of the *Ellipsis*: It will cut the Quadrant of the *Ellipsis* *g e* in *l*, as the Quadrant of the Circle *a e* is cut in *k*.

For drawing the Lines *a h* and *a g*, *k m* and *k l*, to the *Circle* and the *Ellipsis* in the same Plane; the Angles *d c a* and *d c h* being Right, the Plane *a h c* must be perpendicular to the Plane *d c h*; and the Line *h g c* being the common Section of those Two Planes, a Perpendicular from *a* must fall in it; but it must also fall in the Projected *Ellipsis*, by *Prop.* III. *Sect.* I. Therefore in the Point *g* where the *Ellipsis* intersects the Line *h g c*: and, by the Construction, the Line *kp* being drawn parallel to *a c*, and *mp* to *h c*; the Plane *k m p* is perpendicular to the Plane *h c e*, or *h d e*; therefore a Perpendicular from the Point *k* must fall in the Line *m p*; but it must also fall in the *Ellipsis*; therefore at their Intersection in the Point *l* of the Line *m l p*. The same may be prov'd of all the intermediate Points 'twixt *a* and *k*, that they will fall on all the intermediate Points 'twixt *g* and *l*: Therefore the Arch *g l* in the *Ellipsis* represents the Arch *a k* in the *Circle*.

Q. E. D.

Coroll. If the *Ellipsis* is divided into any Number of Parts; to find what Parts of the *Circle* they represent, 'tis evident, that, after having circumscrib'd the *Circle* upon its Longer Axis, and let fall Perpendiculars from the Points of Division in the *Ellipsis* to that Axis, if we continue 'em to the Circumference of the *Circle*, they will cut it as the *Ellipsis* is cut. Thus the Perpendicular *l p* continu'd to *m*, gives the Arch *h m* answering to *g l*, and *m e* answering to *l e*; by the preceding Demonstration.

S E C T.

SECT. IV.

An Example to the foregoing Rules in the Orthographic *Projection of the* Sphere *upon the Plane of the first* Meridian, *or* Solstitial Colure; *to the Latitude of* 51°. 32′ 00″. North. Sect. IV.

IF we concieve the Eye at an Infinite Distance, projecting the Circles of the *Sphere* upon the Plane of the *Solstitial Colure*, with the Principal Ray drawn from it to the Centre of the *Sphere* perpendicular to that Plane; 'tis evident (by *Prop.* II. *Sect.* I.) that the *Solistitial Colure* itself will be a Circle. Which let be the Circle *U h E o*. Fig. 10.

And the *Equator*, *Ecliptic*, *Horizon*, *Equinoctial Colure*, and their *Parallels*, being *perpendicular* to the Plane of the Projection, will be projected in Right Lines equal to their respective Diameters; by *Prop.* I. *Sect.* II. Those representing the *great* Circles passing thro' the Centre of the Projection; and the Representatives of all *Parallels* cutting the Circumference of the Projection in those Points where the *Parallels* themselves cut the *Solstitial Colure* in the *Sphere*.

Let *h* ♎ *o* represent the *Horizon*; the Elevation of the *Equator* above it, to the Latitude of 51° 32′ 00″, is 38° 28′ 00″; the Chord of which Arch to the Radius of the Projection ♈ *p*, laid off from *h* to *æ* above, and from *o* to *q* below the *Horizon*, gives the Points *æ q*, and through which Points and the Centre ♎ of the Projection, if a Right Line be drawn, it will be the Representative of the Equator *æ* ♎ *q*. The greatest Elevation of the *Ecliptic* above the *Equator* is 23 29′ 00″; the Chord of which to the same *Radius* laid off from *æ* to ♋, and from *q* to ♑, gives us ♋ ♈ ♑ the Representative of the *Ecliptic*. And the *Equinoctial Colure* passing thro'

thro' the Poles of the World, and the Centre of the Projection, is represented by the Right Line *p* ♈ *π*, coinciding with the Axis of the *Sphere*, and representing also the Six-a-Clock Hour Circle Morning and Evening; as the upper part of the *Solstitial Colure p a π*, does the Twelve-a-Clock Hour-Circle at Noon, and the under Part of it *p q π* the same at Midnight, in *Astronomical*, or common *English* Hours. The Axis of the *Ecliptic* is ε ♎ ζ having its Poles ε and ζ distant from the Poles of the *Equator p* and π 23° 29′ 00″.

The principal Parallels to the *Equator* describ'd upon the *Sphere* (which may teach us how to draw any other Parallels either to it, or to any other great Circle, perpendicular to the Plane, and projected in a Right Line) are the two *Tropics*, and the two *Polar* Circles; which are projected in Right Lines also; by *Prop*. I. *Sect*. II. and its *Scholium*. And their Representatives intersect the *Solstitial Colure* in the Plane of the Projection in the Points answering to those in which the Circles themselves intersect it in the *Sphere* by *Prop*. I. *Sect*. II. If we therefore lay off the Arch *a* ♋, on the upper part 23° 29′ 00″ from *q* to ♋, and on the lower part of the Projection from *q* to ♑; the Right Lines ♋ γ ♋ and ♑ δ ♑ will represent the two *Tropics*. And the Chord of 23° 29′ 00″ being laid off from *p* to ε and θ, and from π to η and ζ, where the Polar Circles cut the *Solstitial Colure*, that being their Distance from the Poles of the Globe, θ κ ε and ζ λ η will represent the two *Polar* Circles. And after the same manner the Parallels to any other *Perpendicular* great Circle may easily be drawn. And thus much for the drawing of such Circles of the *Sphere* as are represented by Right Lines.

As to Circles *Parallel* to the Plane of the Projection, or the *Solstitial Colure*, they are projected in Circles equal to themselves; by *Prop*. II. *Sect*. I. having

having the Centre of the Projection for their common Centre and Pole. If we know therefore how far distant they are from the Plane of the *Solstitial Colure*, in the Arch of a great Circle perpendicular to both, the *Co-sine* of that Arch is their *Radius*, which is greater or less according to their Distance from, or Parallel Elevation above the Plane of the Projection; by *Prop.* II. *Sect.* II. Having therefore the Semidiameter and Centre of a Circle, the Circle itself may easily be drawn. Thus if a Circle is to be projected upon the Plane of the *Solstitial Colure* elevated 30 Degrees above that Plane, and Parallel to it; the Sine of 60°, or Co-sine of 30° to the *Radius* of the Projection, set off from ♎ to *μ*, gives the *Radius* of the Circle to be projected, which consequently may easily be drawn. Or if we set off the Chord of 60° from *U* to *x* in the Quadrant *U* ♈ *b*, and let fall a Perpendicular from *φ* to the *Horizontal* Line *h* ♎ *o*, it will give the Point *v*, and consequently the *Radius* of the Circle to be projected ♎ *v*.

The *Sun* seeming every Day to run round the *Equator*, or one of its Parallels, in 24 Hours, or equal Spaces of Time, into which the Artificial Day is divided; in one Hour it must run 15 Degrees, in two 30, *&c.*: And the great Circle upon which it is found at any Hour which passes through both the *Poles* of the *Equator* is call'd an *Hour-Circle*: And the *Solstitial Colure* being assum'd as the *First Meridian*, or Meridian of the Place; it will be the Hour-Circle of Twelve at *Noon* and Twelve at *Midnight*: And the *Equinoctial Colure* being at Right Angles, or Perpendicular to it, and passing thro' the Poles of the *Equator*, as all the *Astronomical* Hour-Circles do, it will be the Hour-Circle of *Six* in the Morning and Evening, the *Sun* running a Quadrant of the *Equator*, or one of its Parallels in Six Hours. That Circle is re-

presented here by $p \doteq \pi$. The rest of the Hour-Circles are visibly *Inclin'd* to the Plane of the Projection, and are therefore projected in *Ellipses*; by *Prop.* III. *Sect.* I. having their longer Axis equal to the Diamiter of the Projection, $p \, \Upsilon \, \pi$; and their shorter to twice the Co-sine of their Inclination to the Plane of the Projection, the *Semi-transverse* Axis being the *Radius*.

If we are therefore required to project the Hour-Circle of *Seven* in the Morning, and *Five* in the Evening, *Seven* at Night and *Five* in the Morning, for every one of these Ellipses plainly represents *four* Hours, as the Hour-Circle of *Six* $p \doteq \pi$ is perpendicular to the Plane of the Projection, the next must be 15° *Inclin'd* to it, and consequently 75° elevated above the Plane of the Projection: And consequently the Co-sine of that Elevation, or the Sine of 15°, to the *Radius* of the Projection, set off from $\doteq$ to σ, and from $\doteq$ to ρ, by *Prop.* III. *Sect.* II. gives the shorter Axis of the *Ellipsis* required $\sigma \doteq \rho$. Which being had and the longer Axis $p \, \Upsilon \, \pi$, the *Ellipsis* $p \, \sigma \, \pi \, \rho$, representing the Hour-Circle assign'd may be drawn, by *Coroll.* 2. *Lemma* III. I have here given a Scale (*Fig.* 11.) to the *Radius* of the Projection *Fig.* 10. in which $n \, e$ represents the *Semi-transverse*, and c 15 the *Semi-conjugate* Axis of the *Ellipsis*, representing the Hour-Circle above-mention'd; and 10 d, 20 e, 30 f, 40 g, &c. the Semi-ordinates of that *Ellipsis*, which erected perpendicularly at 10 Degrees distance from each other (which is done by laying off the Chord of 10 Degrees from a to x, and q to z, &c. and drawing the Line x 10 z) give d, e, f, g, &c. thro' which the *Ellipsis* passes. According to the *Corollary* referr'd to. After the same manner $a \, c$ represents the *Semi-transverse* and c 30 the *Semi-conjugate* Axis of the next *Ellipsis*, and upon the Line a 30 the *Semi-ordinates* are mark'd

mark'd out. But these Things must be evident upon bare Inspection to every one who understands the *Corollary* here cited. I shall only here remark, that if the Points where the *Ordinates* are raised are taken to every 5 Degrees or less, the Points thro' which the *Ellipsis* must pass being more and nearer to each other, the *Ellipsis* may be drawn with more Exactness.

These Hour-Circles are Circles of *Longitude* in the Terrestial *Sphere*, and of *Right Ascension* in the Celestial. The Circles of Longitude in the Celestial *Sphere* cut each other in the Poles of the *Ecliptic* ☊ and ☋; and fall in *Ellipses* drawn after the same manner with these I have instanc'd in, having their Requisites deducible from the same Scale. And for determining their *Conjugate* Axes we need not only take the Method now laid down, but if we lay off the Chord of the Complement of the Inclination of any Hour-Circle (for instance) to the Plane of the Projection from *p* on both sides, and let fall Perpendiculars from the Points found to the *Equator* *æ* ♎ *q*, where those Perpendiculars cut the *Equator* they will visibly determine the shorter Axis of the *Ellipsis* requir'd, as they mark out twice the Co-sine of the Inclination of that Circle to the Plane of the Projection: And in the Case of Circles of Celestial Longitude which cut each other in the Poles of the *Ecliptic*, like Chords laid off from ☊ on both sides, and Perpendiculars let fall to the *Ecliptic* ♋ ♈ ♑ will do the same. And if a lesser Circle is to be Projected which is Parallel to one of these *Inclin'd* Circles, its Requisites may easily be found by *Coroll. Prop.* III. *Sect.* II. So that by one or other of these Methods all the Circles of the *Sphere*, whether great or small, may be projected upon the Plane of the *Solstitial Colure*, or that of any other assignable Great Circle; as they are all either

Perpendicular, *Parallel*, or *Inclin'd* to the Plane of any one of the Great Circles of the *Sphere*; and consequently must fall either in *Right Lines*, *Circles*, or *Ellipses*, determinable as to their Natures, Positions, &c. by the Rules laid down in the *Second* Section.

If we are to cut the Representative of a *Perpendicular* Circle into Parts answering any assign'd Division of the Circle it represents, (*viz.* the *Ecliptic*, which, in the Projection we are upon, is represented by the Right Line ♋ ♎ ♑) the Circle required to be drawn about it, by *Prop.* I. *Sect.* III. is the *Solstitial Colure* upon which the Projection is made: If we are therefore to represent the *Twelve Signs* upon it, each of which contains 30°; if we lay off the Chord of 30° from ♋ to Ψ, and from ♋ to χ, and lay a Ruler upon both Points, it will cut the *Ecliptic* in the beginning of ♊ and ♌: The same Chord laid off from ♑ to ω, and from ♑ to *f*; if we lay a Ruler upon both those Points, it will cut the *Ecliptic* in the beginning of ♐ and ♒: The Chord of 60° laid off in the same manner, will enable us to discover the beginnings of ♉ and ♍, ♏ and ♓ and the beginning of ♈ and ♎ are in the Centre of the Projection. And thus may the Representative of any other *Perpendicular* Great Circle be divided at pleasure. And Lesser Circles projected in Right Lines may be divided, as you please, by drawing Circles about them, as the *Solstitial Colure* is drawn about the projected *Ecliptic*; and dividing them as you would have the Projected Circles divided, &c.

Every body must see, from *Prop.* II. *Sect.* III. how to cut a Projected *Parallel* Circle accordingly as that which it represents is cut: I shall not therefore trouble the Reader with any Instance under this Head. And (by *Prop.* III. *Sect.* III.) if

if an *Ellipsis*, the Representative of an *Inclin'd* Circle, is to be cut accordingly as the Circle is which it represents; if we draw a Circle about its Longer Axis, which will be equal to the Circle which it represents, and divide that Circle as the First is divided; and from the Points of Division let fall Perpendiculars to the Longer Axis of the *Ellipsis*; they will cut the *Ellipsis* as the Circle which it represents is cut.

Thus if in the *Ellipsis* p ξ π ♄, we are to cut from off the Quadrant p ξ or p ♄ an Arch answering to 55° of the Circle it represents; as the Circle p α π q is drawn about its Longer Axis, and equal to that Circle; if we set off the Chord of 55° to the *Radius* of the Projection from p to ♂, and from p to ♃, and draw the Line ♂ ♃, it will cut the Quadrant p ξ in ♀, and p ♄ in ☿, as the Circle it represents is cut: And p ♀ and p ☿ will answer to 55° in the *Nine* and *Three-a-Clock* Hour-Circle Morning and Evening, counted from the North Pole of the Globe. The perpendicular ♂ ♀ ☿ ♃ will also cut the rest of the Hour-Circles after the same manner; as any one must see, who understands the *Third* Problem of the *Third* Section. And if Lesser *Ellipses*, Representatives of Lesser *Circles*, are to be divided, the same Method is to be taken: The Demonstration of that *Problem* not taking their being Greater or Less into Consideration, but equally concluding whether they are Great or Small.

In the Projection before us, a great many more Representatives of other Circles of the *Sphere* might have been drawn; but they would have rendered it confused, and therein less intelligible to such as are not already acquainted with this kind of Projections. By the Rules already laid down, those who have a mind to draw more may be supply'd with Requisites to draw them;

and from the Instances now given, they are taught how to reduce those Rules to Practice. And whereas I might add to this several other Instances of the *Orthographic* Projection of the *Sphere* upon several other of its Great Circles, they being infinite, and each of 'em admitting a Projection upon it: Yet, from the Instance already given, the Reader may easily learn how to project the *Sphere Orthographically* upon any other Plane: And the Difficulty of Drawing *Ellipses* exactly, making this kind of Projections the less used, I shall produce Instances upon more Planes, when I come to the last Section of the next Chapter, relating to *Stereographick Projections*.

CHAP.

CHAP. II.

Of the STEREOGRAPHIC Projection of the SPHERE.

SECT. I.

Of the Appearance of the Circles of the Sphere *upon the Plane of the Projection.* Sect. I.

PROP. I. Theorem.

All Circles which pass through, and cut each other in the Poles of that Great *Circle upon the Plane of which the Projection is to be drawn, being Perpendicular to that Plane, are represented by Right Lines upon the Plane of the Projection.*

DEMONSTRATION.

Conceive *a c b* to be a Diameter of the Circle upon the Plane of which the Projection is to be made, which may, for Brevity's-sake be call'd the *Primitive* Circle, whose Plane stands at Right Angles to that of the Circle *a e b d*: Its Poles being 90° distant from its Circumference, are represented by the Points *e* and *d*. Let the Eye be at *e*, as is requir'd in *Stereographic* Projections; and as its Axis is in the same Plane with the Circle *a e b d*, it must view the Arch *a d b* in the Plane of its own Circle; and it must view it in the Plane of the *Primitive* Circle at the same time; therefore in their Common Intersection: But the Common Inter- *Fig.* 12.

Intersection of any Two Planes is a Right Line; therefore the Representative of the Semi-circle *a d b* passing thro' the Pole *d* of the *Primitive* Circle is a Right Line. The same may be demonstrated of the Semi-circle *a e b*, which will be represented by the Right Line *a c b*.

Q. E. D.

LEMMA I.

If a Cone be cut by a Plane Parallel to the Plane of its Circular Base, the Circumference of that Section will be a Circle.

Demonstration.

Fig. 13. LET the Cone *a f c e d* be cut by a Plane *i h k l* thro' the Axis *a b*, then will the common Intersection of this Plane, with the Plane *i k h l* Parallel to the Base *f d e c* be the Right Line *l g k*: Which Right Line *l g k* will be Parallel to the Diameter of the Base of the Cone *f b e* and both will lie in the same Plane *f a e*; therefore (by the 4th of the 6th of *Euclid*) it will be *a b* : *b f* :: *a g* : *g l* and as *a b* : *b e* :: *a g* : *a k* wherefore by the (16 of the 5th of *Euclid*) it will be *b f* : *b e* :: *g l* : *g k*; but *b f* is equal to *b e*, therefore *g l* is equal to *g k*. Again, to any other Point as *d*, in the Circumference of the Base of the Cone from the *Vertex a* draw the Right Line *a d*; this being in the Superficies of the Cone shall cut the Plane thro' *i k h l* somewhere as in *i*, and draw *g i*, which will be Parallel to *b d* and in the same Plane with it: Wherefore, as *a b* : *b d* :: *a g* : *g i* therefore whereas *b d* equal to *b e* is equal to *b f*, *g i* equal to *g k* is equal to *g l*, and the same will hold good where-ever the Point *i* falls, wherefore the Section

Section $i\ k\ h\ l$ will be a Circle and g its Center, by the 15th and 16th Def. of the 1st of *Euclid*.

Q. E. D.

LEMMA II.

If a Scalene Cone a I m k n *be cut by a Plane through its Axis, at Right Angles with its Base, so as to make the Triangle* a l k : *and from this Triangle a smaller be cut off* a e f *similar to the former, but subcontrarily posited*; (i. e. *so that the Angle* a e f *be equal to the Angle* a k I) *and the Cone be cut by a Plane according to the Direction of* e f, *at Right Angles with the Plane of the Triangle* a k I; *the* Section *form'd in the Surface of the Cone* e h f g *will be the Circumference of a Circle.* Fig. 14.

DEMONSTRATION.

THrough any assign'd Point b of the Line $e\ f$ draw the Line $c\ d$ Parallel to the Base $k\ I$; thor' which let a Plane pass perpendicular to the Triangle $a\ k\ I$, and Parallel to the Plane of the Base; it will make the *Section* in the Surface of the Cone $d\ b\ c\ g$ the Circumference of a Circle, by the preceding *Lemma*; cutting the Circumference of the Plane of subcontrary Position in the Points h and g; and the Plane it self in the Line $h\ b\ g$. Now the Figure $g\ c\ h\ d$ being a Circle, the Square of $b\ g$ will be equal to the Rectangle under $c\ b\ d$ by *Lemma* II. *Cap.* I. $b\ g$ being perpendicular to $c\ d$ by the Construction. And because of the similar Triangles $e\ b\ c$ and $d\ b\ f$, it holds $e\ b : c\ b :: b\ d : b f$; and $e\ b \times b\ f = c\ b \times b\ d = \square\ b\ g$: Therefore the Point g in the Figure $g\ e\ b\ f$ is in the Circumference of a Circle. The same might be

be demonstrated of the Point *h*, and all the Points in it. Therefore the *Section* is a Circle.

Q. E. D.

PROP. II. Theorem.

All the other Circles of the Sphere, *whether great or small, which pass not through the Poles of the* Primitive *Circle upon the Plane of which the Projection is drawn, are Circles upon the Plane of the Projection.*

Demonstration.

Fig. 15. IF the Circle is *Parallel* to the Plane of the Projection it is a lesser Circle, and projected into a Circle by *Lemma* I. If it is *Inclin'd*, let the Eye be at *a*, projecting the *Inclin'd* Circle *c b* upon the Plane of the Primitive *h k g* continued as far as requir'd: 'Tis plain that *c b* is projected into *e f*: And the Triangles *a b c* and *a e f* are subcontrary; and consequently as *c b* is a Circle, its Projection *e k f* will be a Circle also. For $a\,c\,d = a\,k\,e$, and *k a e* is common; therefore $a\,e\,k = a\,d\,c$. But (by 21. III. *Euclid*) $a\,d\,c = a\,b\,c$; therefore $a\,b\,c = a\,e\,f$; and, *b a c* being common, $a\,f\,e = a\,c\,b$; that is, *a b c* and *a e f* are subcontrary; and *c b* representing a Circle, *e f* will represent one also, by *Lemma* II.

If the Circle to be projected is *perpendicular* to the Plane of the Projection, and passes not through its Poles; it will be a Circle for the same Reason. Let the Circle be represented by *i b*, perpendicular to the Plane *h k f*; it is projected by the Eye at *a* into *l f*: And the Angle *a i b* being equal to *a c b*, by 21. III. *Euclid*; and $a\,c\,b = a\,f\,l$, as is already prov'd; therefore $a\,i\,b = a\,f\,l$: But *b a l* is common to both, therefore $a\,l\,f = a\,b\,i$: Therefore the Diameters *i b* and *l f* are subcontrarily posited, and

and the Planes which they represent; and *i b* being the Diameter of a Circle, *l f* will be so too; in the Cone cut by the Triangle *a i b* continued as far as requisite.

Q. E. D.

Scholium. That this Proposition, which is one of the great Fundamentals of *Stereographic* Projections, may be throughly understood by every Reader, it must be consider'd; that here, as well as elsewhere, I suppose the Circles to be projected, and the Plane of the Projection, standing at Right Angles to the Plane of the Circle drawn upon the Paper *a g d k*. And if we conceive Lines drawn from every Point of the Circumference of a Circle *parallel* to the Plane of the Projection, thro' that Plane to the Eye; or, if the Circle is nearer the Eye than the Plane, if we suppose them drawn to the Eye, and then continued to the Plane of the Projection, 'tis evident that they will mark out a Circle in that Plane: As in *Fig.* 13, Lines drawn from every Point of the Circumference of *e d f c*, to the Eye at *a*, form in the Plane of the Projection, to which *e d f c* is *parallel*, the Circle *k h l I*: Or, if *k h l I* is to be projected, as those Lines, drawn to the Eye at *a*, and continued to the Plane mark out the Circle *e d f c*. If we farther conceive Lines drawn from every Point of the Circumference of the Circle whose Diameter is *c d*, in *Fig.* 15. to the Eye at *a*, they will evidently form a Scalene Cone, answering to the Cone *a I m k n* in *Fig.* 14. and this Cone will be cut subcontrarily by the Line *e k* in the Plane of the Projection, as the other (*Fig.* 14.) is by the Line *e f*: For *e k a* = *a c d*, and *k a e* is common to *d c a* and *e k a*; therefore *a e k* = *c d a*. Supposing therefore upon *e k*, the Representative of *c d* in the Plane of the Projection, a Plane to cut the Cone, as

Vid. Schol. *ad* Def. 10.

e h

e h f g in *Fig.* 14. cuts the Cone *a I m k n*, perpendicularly to the Plane of *a g d h*, as the Projection of the Circle represented by *c d* must in the Plane of the Projection; by its subcontrary Position to the Circular Base of the Cone (*viz.* the Circle to be projected) it must be a Circle; as in *Fig.* 14, the subcontrary Position of the Figure *e g f h* to the Circular Base *I m k n* makes it a Circle. And so for all other Circles whatever which pass not thro' the Poles of that Circle upon the Plane of which the Projection is drawn.

SECT. II.

Sect. II. *Of the Methods for determining the Requisites for the Delineation of the rest of the Circles of the* Sphere *upon the Plane of the Projection.*

PROP. I.

If a great Circle which is Perpendicular *to the Plane of the Projection is to be projected; as it passes through the Poles of the Projection, it must be done by drawing a Right Line through the Centre of the Projection, and the two Points in which that Circle cuts its Circumference; infinitely on each side.*

DEMONSTRATION.

Fig. 16. LET the *great* Circle *b d c e*, passing through the Poles of the Projection *b* and *c*, be projected upon the Plane of a *great* Circle whose Diameter is *d e*, by the Eye at *b*, being Perpendicular to that Plane. It is projected into a Right Line, by *Prop.* I. *Sect.* I. passing through the Center of the Projection *a*, because the Poles of the Projection

ction *b* and *c*, thro' which it passes, are projected in that Centre. And in this Right Line, *a t* and *a s* visibly represent the Arches *c q* and *c r*; *a d* and *a e* the Arches *c d* and *c e*: And the projected Right Line must pass thro' the Points *d* and *e*, where the Circle *b d c e* intersects the Plane of the Projection at Right Angles with it; because those Points are view'd by the Eye at *a* in the Planes of both the Circles, and consequently in their common Intersection. Farther, If more of the Circle is projected, the Right Line *e a d* must be continued on each side; and then *d f* and *e g* will represent the Arches *l d* and *m e*; *d I* and *e h* those of *k d* and *n e*. And if the whole is to be projected to the Point *a*, the Angles *I b a* and *h b a* will visibly so increase 'till they become *Right*, the Lines *b I*, *b h* coinciding with *b p*, *b o*: And *b p*, *b o* being Parallel to *a I*, *a h* they will intersect 'em no where; but *a I*, *a h* will become Infinite.

Q. E. D.

Coroll. 1. Hence we learn the First Fundamental Difference 'twixt *Orthographic* and *Stereographic* Projections: For whereas in *Orthographics* a Perpendicular great Circle is projected, by the Eye at an Infinite Distance, into a Right Line equal to its Diameter: In *Stereographics*, when the Eye is in the Surface of the *Sphere*, it is projected into one infinitely extended both ways from the Centre of the Projection. Which proves that these two sorts of Projection are specifically different. As *Scenographics*, or common *Perspective*, is from them both; projecting the Line longer or shorter in Proportion to its Nearness to, or Remoteness from the Surface of the *Sphere*. And 'tis manifest that there can be no more Kinds of Projection, which will represent the whole of any Object upon the same Plane, than these *Three*:

Because

Becauſe ſuppoſing the Eye within the Surface of the *Sphere*, for inſtance, at the Point *v*, it cou'd project no more of the Circle *b d c e* upon the Plane *d a e* continued, than the Arch cut off by the Right Line *x v z*.

Coroll. 2. Hence alſo we are taught what *Perpendicular* great Circle is repreſented by any projected Right Line, knowing the Points where that Line interſects the Plane of the Projection: For there the great Circle projected interſects that upon the Plane of which the Projection is made.

PROP. II.

Fig. 17. *If a leſſer Circle whoſe Diameter is* h i, Perpendicular *to the Plane of the Projection, whoſe Diameter is* c a d *continued as far as requiſite, is to be projected upon that Plane; the Centre of its Repreſentation upon the Plane of the Projection ſhall be in the Line* c a d f, *call'd the Line of Meaſures, diſtant from* a *the Centre of the* Primitive *Circle upon which the Projection is made; by the Secant of that leſſer Circle's Diſtance from its Pole; and its Semi-diameter ſhall be equal to the Tangent of that Diſtance.*

DEMONSTRATION.

LET the leſſer Circle *h i* be projected upon the Plane of the great Circle *c d*, by the Eye at *g*; 'tis plain that the End of its Diameter *h* is projected in the Line of Meaſures at *k*, and *i* at *f*; and *k f* biſſected, gives *e* the Centre of the Circle to be projected; and *e k* or *e f* its *Radius*: Draw the Lines *a b*, *h f*, and *h e*. The Triangles *a b f* and *g h b* being Rectangular, and having the Angle at *b* common, are Similar; therefore *b g h* = *b f a*: But (by 20 III. *Euclid*) *b g h* = $\frac{1}{2}$ *b a h*,

bah, and $afb = \frac{1}{2} hea$, therefore $bah = hea$: But $bah + hae$ make a Right Angle, therefore $hea + hae$ make a Right Angle; and consequently ahe is alſo a Right Angle: by 32. I. *Euclid.* And ae the Diſtance of the Centres, is the Secant of the Arch hd, the Diſtance of the Circle to be projected from its Pole d, to the *Radius* of the Projection ad or ah: And he the Semi-diameter of the Circle to be projected is the Tangent of the ſame Arch.

Q. E. D.

Coroll. Hence we learn how to derive any Perpendicular leſſer Circle from its Repreſentative upon the Plane of the Projection: For if that part of it only is projected which falls within the great Circle upon whoſe Plane the Projection is drawn; if we compleat the Circle by 5. IV. *Euclid*, its *Radius* is equal to the Tangent of its Diſtance from the neareſt Pole of the great Circle to which it is Parallel to the *Radius* of the Projection. Or if we biſſect the Arch of the Projection intercepted by this Circle we have its neareſt Pole; and the Chord of the Arch intercepted 'twixt that Pole and either Point of Interſection is the Chord of the Compliment of its Diſtance from the Great Circle to which it is parallel; to the Radius of the Projection.

PROP. III.

If a Great Circle Inclin'd *to the Plane of the Projection is projected upon the Plane of another Great Circle; the Centre of the Circle to be projected is diſtant from the Centre of the* Primitive *Circle, in the common Interſection of the Plane of a Great Circle at Right Angles to the Planes of thoſe two Great Circles, and the Plane of the Projection,*

(call'd the Line of Measures*) by the Tangent of the Elevation of that Great Circle above the Plane of the Projection, to its* Radius: *And the* Radius *of the Circle to be projected is equal to that Distance, and the Tangent of half the nearest Distance of that Circle from the Pole of the Projection opposite to that in which the Eye is plac'd, taken together.*

DEMONSTRATION.

Fig. 18. CONceive a Great Circle as *b c*, *Inclin'd* to the Plane of another Great Circle as *f g*, upon which it is to be projected by the Eye at *e*; both being imagin'd perpendicular to the Plane of *d f e g*: I say the Centre of the Circle to be projected will be found distant from that of the *Primitive a* in the Line *f g h* (which is the common Intersection of the Circle *d f e g* at Right Angles to the two other Circles with that of the Projection; and of the Representative of the Circle to be projected; and is call'd the *Line of Measures*) by the Tangent of the Elevation of that Circle above the Plane of the Projection; and its Semi-diameter will be Equal to that Distance, and the Tangent of $\frac{1}{2}$ *d a b*, taken together. For the Extremity *b* is projected in the Line of Measures at *l*, and *a l* is the Tangent of $\frac{1}{2}$ *d a b* to the *Radius* of the Projection; by 20. III. *Euclid.* And the Extremity *c* is projected at *h*; and *l h* is bissected at *i*, which is therefore the Centre of the Circle to be projected: And it is distant from *a* by *a i* the Tangent of *a e i* = *b a f*: For the Triangle *l e h* being Right Angled at *e* as that Angle is in a Semi-circle, and having the Angle at *l* common with the Triangle *a l e*, which is also Right-angled at *a*, they are Similar; and *l e a* = *i h e* = *i e h*, as *i e* and *i h* are *Radii* of the Representative of the Circle to be projected, and consequently

consequently ieh an *Isosceles*: But $eba = lea$; therefore $bea + eba = bad$ are Equal to $ieh + ihe = aie$; therefore $bad = aie$; and consequently $aei = baf$: And ai is the Tangent of that Angle to ae the *Radius* of the Projection: And $ai + al = il$ the *Radius* of the Representative of the Circle to be projected.

Q. E. D.

Scholium. In this Demonstration one thing is taken for granted which perhaps every one may not see the Truth of at first Sight, tho' it be very manifest to those who understand the Nature of the *Sphere*: It is this, that ie is Equal to il or ih; the *Radius* of the Representative of the *Inclin'd* Circle whose Diameter is bc, in the Plane of the Projection. And that it is so we shall clearly see if we supply in our Minds the Circle to be projected whose Diameter is bc, and the Plane of the Projection whose Diameter is fg, and conceive 'em both at Right Angles to the Plane of the *Great* Circle $dfeg$; and to these add the Circle whose Diameter is ed at Right Angles to the same Plane. For as all the *Great* Circles of the *Sphere* cut each other at a Semi-circular Distance, by *Coroll.* 2. *Def.* 2; and their Representatives in the Plane of the Projection do the same, by *Coroll.* 2. *Def.* 3: Hence it is plain that the Representative of the Circle whose Diameter is cb must cut the Plane of the Projection whose Diameter is fg on each side at a Quadrantal Distance from the Point f; but so must the Circle whose Diameter is de, as de and fg are at Right Angles to each other; therefore the Projected Circle must cut the Circumference of the Plane of the Projection in the same Point in which it is cut by the Circle whose Diameter is de: But a Line drawn from the Centre of the projected Circle to that Point of Intersection is

is Equal to its *Radius* *i l* or *i h*; it is alſo Equal to a Line drawn from the ſame Centre to any Point in the Circumference of the Circle whoſe Diameter is *d e*; as *i a* is at Right Angles to its Plane, and conſequently *i e* conceiv'd as fix'd in the *Vertex* *i*, and turn'd round the Circumference of the Circle whoſe Diameter is *e d* will generate a *Right Cone*, whoſe *Vertex* is equi-diſtant from every Point in the Circumference of its *Baſe*: Therefore *i e* is Equal to the *Radius* of the projected Circle *i l*, or *i h*. Or, If we conceive the Circle *d f e g* to be that upon the Plane of which an *Inclin'd Great* Circle is to be projected, as *d l e h*; their Points of Interſection being at a Quadrantal Diſtance from their projected Points of greateſt Inclination *l* and *f*, *l f* being the projected Meaſure of their Inclination, or of the Angles *l d f* or *l e f*, they muſt be at *d* and *e*; and where the *Inclin'd* Circle interſects the Primitive, there its Repreſentative muſt interſect it alſo: Therefore a Right Line drawn from the Centre of the Repreſentative to either Point of Interſection, as *i e*, muſt be its *Radius*, or equal to *i l* or *i h*.

Coroll. Hence we are taught what *Inclin'd* Great Circle is repreſented by any one drawn upon the Plane of the Projection: For having its Centre, or finding it if you have it not, and opening your *Sector* to the *Radius* of the Projection, the Diſtance of that Centre from that of the Projection taken 'twixt your Compaſſes will give you upon the Line of Tangents the Inclination of the Circle repreſented to the Plane of the Projection. Or the *Radius* of the projected Circle is the Secant of that Inclination, &c.

PROP.

PROP. IV.

If a Lesser Circle, whether Parallel, *or* Inclin'd, *to the Plane of the Projection, is to be projected upon the Plane of a Great Circle; It is done by laying off from the Centre of the Projection, in the Line of Measures, the Tangent of Half the Distance of each Extremity of its Diameter from the Pole of the Projection, opposite to that in which the Eye is plac'd; and the Distance of these Two Projected Extremities bissected, will be the Centre of the Representative of that Circle.*

Demonstration.

LET *d c* be projected, its Representative is *l k*, the Eye being at the Pole *e* of the Plane of the Projection *f a g*, continued each Way as far as requir'd; of which *a k* is the Tangent of Half the Distance of *d* from the Pole *b*; and *a l* of Half the Distance of the Point *c* from the same, by 20. III. *Euclid*, both in the *Inclin'd* Circle *c d*, and in the *Parallel d c*. And each *l k* bissected, gives the Centre of the Representative of the Circle to be projected. *Fig.* 19.

Q. E D.

Corollary 1. Hence, having a Representative upon the Plane of the Projection, we may easily discover what *Parallel* or *Inclin'd* Lesser Circle it represents: For if it is *Parallel*, (in which case its Centre coincides with that of the Projection) and its *Radius* is Greater than that of the Projection, then the Circle it represents is 'twixt the Eye and the Plane, and its *Radius* is equal to the Tangent of Half its Distance from the Pole of the Projection opposite to that in which the Eye is plac'd,

to the *Radius* of the Projection: If its *Radius* is less than that of the Projection, then the Circle it represents is 'twixt the Plane and the opposite Pole; and its *Radius* is equal also to the Tangent of Half its Distance from that Pole: Which being had, its Distance from the Plane of the Projection may be had also. If the Representative has its Centre different from that of the Projection, the Circle it represents is *Inclin'd* to the Plane; as *c d* is, 'twixt the Plane and the Pole *b*, having *m* for its Centre: And the Difference betwixt half its nearest and greatest Distance from the Centre of the Projection or Pole opposite to that in which the Eye is placed, gives us its *Inclination* to the Plane of the Projection; which being had, we have the Great Circle to which it is Parallel being equally inclin'd to the same Plane; and if we project that Circle, and take it for a New Plane, the Radius of the Inclin'd Lesser Circle is Equal to the Tangent of Half its Distance from the Pole of the New Plane, opposite to that in which the Eye is plac'd, or the Centre of the New Projection *m*, to the *Radius* of that New Projection. These Things are so plain, that they need no particular Illustration.

Coroll. 2. Hence we see, that whereas in *Orthographic* Projections, each Parallel Representative represents Two Parallels to the Plane of the Projection in the *Sphere*, which are equidistant from the Plane on each Side of it; in *Stereographics*, every Parallel in the *Sphere* has its particular Representative: Those 'twixt the Eye and the Plane being projected much larger than Equidistant Parallels 'twixt the Plane and its Pole, opposite to that in which the Eye is plac'd; the former falling always in the Plane continu'd beyond the *Radius* of the Primitive Circle, and the latter, within the Plane of

of the Projection. And whereas each *Ellipsis*, Representative of a Lesser Circle *Inclin'd* to the Plane of the Projection, represents a Parallel Lesser Circle Equidistant on the contrary Side from the Plane of the *Great* Circle to which the First is Parallel, in *Orthographic* Projections; as every one must see, who understands *Prop.* III. *Sect.* I. *Cap.* I. In *Stereographics*, every *Inclin'd* Lesser Circle has its peculiar Representative, for the same reason that every *Parallel* Lesser Circle has. Which is one of the great Advantages of *Stereographic* above *Orthographic* Projections, with respect to their Representatives of the Circles of the *Sphere* upon the Plane of the Projection. Another of great Moment being this, that *Inclin'd* Circles are projected into Circles in the former; whereas in the latter, they are projected in *Ellipses*; which are not drawn, without great difficulty, so exactly as to solve any *Problems* of the *Sphere* with any tolerable Accuracy.

Scholium 1. Whereas sometimes we have occasion to take a Projected Circle for a *Primitive*; and to project other Great Circles upon that, under different Elevations; intersecting it and each other in Two assign'd Points or Poles, for Instance, in the Line of Measures of the *Primitive* Circle; this may easily be done by the Third *Proposition* of this *Section*, without calling in any New Principles. For in the Scheme annex'd, after we have projected the Circle *b d g h e*, elevated 40° above the *Primitive c d e n*; if we be requir'd to project any Number of Great Circles upon the Plane of this projected Circle, passing thro' the Points *b* and *l*; it is but completing this Circle *b d g h e*; and then (by the Third *Proposition* of this *Section*) the Tangent of the *Inclination* of the Circle to be projected, (for Instance, 40°) to the *Fig.* 20.

Plane of the *Projected Primitive*, laid off in the New Line of Measures *g h m*, to the New *Radius f h*, or *f b*, from *f* to *i*, gives *i* the Centre of the Circle to be projected; and the Tangent of 25 Degrees, *viz.* of Half the Distance of the Circle to be projected from the Pole of the *Projected Primitive* opposite to that in which the Eye is plac'd, gives *f k*; which with *f i* gives the *Radius* of the Circle to be projected, intersecting the *Projected Primitive* at *l* and *b*; and when compleated, making the Circle *l k b n m*. After the same manner may infinite Centers be found in the Line of Measures *g f h* continu'd both Ways from the Centre *f*, for Circles cutting each other in the Points *b* and *l*; under whatever Degrees of Inclination, and the *Radii* completed, and themselves drawn.

Schol. 2. By the *Two* following *Theorems* also, the Rules for the Projection of all Great or Lesser Circles, which any-way intersect the Plane of the Projection, may be demonstrated; as any One may see, who will apply 'em to the Operation in the preceding *Scholium*; or to what *Examples* shall hereafter follow. But as they only demonstrate the Thing when done, without leading us into the Discovery of the Rules for Operation *a priori* at the same time; I chose rather to deduce the Requisites for *Stereographic Projections* from the *Four* foregoing *Propositions*.

THEOREM I.

In Stereographic *Projections, the Angles made by the Circles on the Surface of the Sphere, are Equal to those made by their Representatives on the Plane of the Projection.*

Fig. 21.

DEMON-

DEMONSTRATION.

SUppose the Eye at *a* projecting the Angle *l e m* upon the Plane represented by *k f m* at Right Angles to the Plane of *c k a m*: Draw *e h* and *e i* Tangents to the Two Circles by which the Angle is form'd, from the Point of Intersection *e*: The Planes of *k f m* and *h e i* are both perpendicular to that of the Circle *c k a m* by the Construction; therefore their common Intersection *h i* is perpendicular to the Right Line *k f m* continued. And the Eye at *a* projecting the Tangents *e h* and *e i* in *f h* and *f i*; draw *d e* parallel to *k m*, and join the Points *a* and *d*. The Angle *h e a* is equal to *a d e*, by 32. III. *Euclid*; which is equal to *a e d*, the Triangle *a e d* being, by the Construction an *Isosceles*; and *a e d* = *h f e*, by 27. I. *Euclid*, which is therefore equal to *h e f*; and *e h* = *f h* by 6. I. *Euclid*. Therefore in the Triangles *h e i* and *h f i*, *h e* and *h f* being equal, *h i*, common to both, and the Angles at *h* Right, they are equal in all respects; and *h f i* = *h e i* by 4. I. *Euclid*.

Q. E. D.

THEOREM II.

If Two Circles cut each other upon the Plane of the Projection, the Angle form'd by them at their Intersection is equal to that form'd by Radii *drawn from their Centres to the Point of Intersection.* Fig. 24.

DEMONSTRATION.

DRaw Lines from the Centres *a* and *d* to the Point of Intersection *g* : and Tangents to each Circle at the same Point: Because the Evanescent Parts of the Tangents *g c* and *g f* next the Point of Intersection *g*, coincide with the

the infinitely small Portions of the Circles at the same Point, they will have the same Direction; and consequently, the Angle cgf is equal to the Angle bge: But dgf and agc are Right Angles; and consequently, the common Angle dgc taken from both, makes $dga = cgf = bge$.

Q. E. D.

Corollary. Hence we have a Demonstration of the Method of Projecting *Inclin'd* Great Circles discovered in the *Third* preceding *Proposition*, and of that of Projecting others upon a projected Plane cutting each other in a given Point, deduc'd in the *First Scholium* to the *Fourth Proposition.* For as in *Stereographic* Projections, the Angles made by the Circles on the Surface of the *Sphere*, are equal to those made by their Representatives upon the Plane of the Projection; 'tis plain, that if we project one Great Circle so upon the Plane of another, as that these Angles shall be equal, we
Fig. 20. project it aright: If we have therefore the *Inclin'd* Circle $bglh$ to project upon the Plane of $cdfe$, Inclin'd to that Plane 40°, we must make the Angle ben 40°: Which is done by laying off the Tangent of 40° from a to f, to the *Radius* ac or ae, and drawing the Circle $bglh$; for $ben = aef = 40°$, by *Theor.* II. And if $kbml$ *Inclin'd* also 40° to $bglh$, is to be projected upon it, intersecting it in the Points b and l, the Tangent of 40° to the *Radius* fl, laid off from f to i, gives its Centre and the Circle it self $kbml$ *Inclin'd* 40° to $bglh$; for $klg = fli$, $= 40°$, by the same *Theorem*, &c.

SECT.

SECT. III.

Of the Mensuration of Circles projected Stereographically, *whether* Great *or* Small. Sect. III.

PROP. I. Problem.

If a Circle is projected into a Right Line, which a Great Circle can only be in Stereographic *Projections, to cut that Line into Parts Representative of any assign'd Divisions of that Circle.*

Solution.

LET the Right Line *i a c b g* infinitely continued on each Side from the Point *c* be the Representative of the Circle *a d b e*; and let it be requir'd to divide the Line *i a c b g* as the Circle *a d b e* is divided, in the Points *l*, *f*, *h* and *k*: 'Tis plain that a Ruler laid upon the Points *e* and *l* will cut off *c m* representing the Arch *d l*; one from *e* to *f* will cut off *c g* representing the Arch *d b f*, &c. Or the Tangent of half the Arch *d l* to the *Radius* of the Projection laid off from the Point *c*, will give *c m* Representative of *d l*, and that of half the Arch *d b f* from the same Point will give *c g* Representative of *d b f*, &c. For *c m* is the Tangent of *c e m* $= \frac{1}{2}$ *d c l*, and *c g* of *c e g* $= \frac{1}{2}$ *d c f*, to the *Radius* *c e*, by 20. III. *Euclid*. Fig. 23.

Q. E. F.

Corollary. If a Right Line Representative of a Great Circle *perpendicular* to the Plane of the Projection is cut into any Number of Parts, we may know what Arches of that Circle are represented by any of those Parts by the Converse of this *Problem*: Thus opening your *Sector* to the *Radius*

c e,

$c\,e$, $c\,m$ is the Tangent of half the Arch which it represents, *viz.* $d\,l$: And $c\,i$ the Tangent of half the Arch which it represents, *viz.* $d\,a\,h$.

LEMMA I.

If an Inclin'd *Great Circle is to be projected upon the Plane of another Great Circle, the Pole of the Circle to be projected, which falls within the Primitive, is distant from the Centre of the Plane of the Projection, by the Tangent of half the* Inclination *of the Circle to be Projected to the Plane of the Projection: And that which falls without the Primitive is distant from the same Centre by the Tangent of the Complement of half that* Inclination *to* 90 *Degrees.*

DEMONSTRATION.

Fig. 24. IN the Scheme annex'd let $a\,c$ represent the Plane of the Projection, $e\,f$ the Circle to be projected, d the Pole of the *Primitive* in which the Eye is plac'd, b its opposite Pole; g and h the two Poles of the Circle to be projected; $g\,h$ being at Right Angles to $f\,e$. Therefore,

1. *First*, m represents the Pole g; but $m\,d\,k = \frac{1}{2}\,g\,k\,b$, by 20. III. *Euclid*; and $g\,k\,b = e\,k\,c$, as each of 'em added to $b\,k\,e$, makes a Right Angle; therefore $m\,d\,k = \frac{1}{2}\,e\,k\,c$, the *Inclination* of the Circle $e\,f$ to the Plane of $a\,c$: But $m\,k$ is the Tangent of $m\,d\,k$ to the *Radius* of the *Sphere*; therefore (k being the Representative of the Pole b) $m\,k$, the Distance of the two projected Poles m and k, is the Tangent of half the *Inclination* of the Circle to be projected to the Plane of the Projection.

2. *Secondly*,

2. *Secondly*, *n* represents the Pole *h* which falls out of the Projection; and in the Triangle *d g h* the Angle *g d h* is right; by 31. III. *Euclid*; and the Triangles *m d n* and *m k d* having the Angles *m d n* and *m k d* Right, and the Angle at *m* common, the Angles *m d k* and *k n d* must be Equal; therefore *d k n* being Right, *k d n* is Equal to *k m d*, the Complement of *k d m* (= ½ *e k c*) to 90 Degrees: But *k n* is the Tangent of *k d n* to the *Radius* of the *Sphere*; therefore *k n*, the Distance of the two projected Poles *k* and *n*, is Equal to the Tangent of the Complement of half the *Inclination* of *e f* to the Plane of the Projection.

Q. E. D.

Corollary. The Poles of every Lesser Circle, as *a l*, are the same with those of the Great Circle to which it is parallel, *viz.* of *e f*; i. e. *g* and *h*: Knowing therefore the *Inclination* of a Lesser Circle to the Plane of the Projection, we know the *Inclination* of its Parallel Great Circle, two Parallel Planes being equally Inclin'd to a third Plane, and consequently may determine its Poles by the precedent *Lemma*.

LEMMA II.

If a Plane be drawn thro' the two remotest Poles of two Equal Circles of the Sphere, *it will form a Circle in the Surface of the* Sphere, *which moving upon a Line connecting those two Poles as an Axis will cut off equal Portions, betwixt itself and a great Circle to which those Circles are perpendicular, from the Peripherys of those Circles.*

DEMONSTRATION.

IN the Scheme annex'd, the Two remotest Poles of the Circles *a f d* and *g f e* are *b* and *h*; and the Circle *h l k b* passing through those Poles cuts off Fig. 25.

off equal Arches of those Circles 'twixt itself and $bahd$ to which they are supposed Parallels, $al = gk$, and consequently $lf = kf$. For, by the Nature of the *Sphere*, the Arch bkl which reaches from the Pole to the Circumference of the Circle afd is equal to the Arch hlk from the Pole to the Circumference of an Equal Circle gfe; therefore their Subtenses bl and hk are Equal: And kl being common to Two Equal Arches bl and hk, bk and hl are equal, and consequently $bhk = hbl$; by 27. III. *Euclid*. Farther, In the Triangless hcm and bcn the Angles at c are right by the Nature of the *Sphere*; and $cbh = chb$ by *Isosceles* Triangles, and $ch = cb$; therefore they are Equal in all respects, and $bn = hm$, and cn to cm; by 26. I. *Euclid.* Therefore in the Triangles lnb and kmh, the Sides lb and kh, bn and hm being Equal, and $lbn = khm$; they are Equal in all respects; and $ln = km$; by 4. I. *Euclid*. And in the Triangles cmk and cnl, the Sides cm and cn, mk and nl being prov'd Equal; and $ck = cl$ as *Radii* of the same, or Equal Circles; they are Equal in all respects; by 8. I. *Euclid*; and consequently $kcg = lca$, and $gk = al$; and their Complements to a Quadrant $kf = lf$.

Q. E. D.

Fig. 26. *Scholium.* Tho the precedent Scheme relates only to great Circles, yet the Demonstration is Universal, and holds in all equal Circles, which are consequently equally distant from their respective Poles. For (in *Fig.* 26.) let the Lesser Circles qkr and gle be Equal, and let the Circle $hlkb$ pass thro' their remotest Poles b and h: The Arches bk and hl must be Equal; *viz.* the Distance of the two Circles from their respective nearest Poles, and kl being added on both sides, bl must be Equal to hk; and consequently the Chord bl to the

the Chord $h\ k$. Farther, In the Triangle $b\ c\ h$ the Angles at b and h are equal by *Isosceles* Triangles; and in the Triangles $b\ o\ m$ and $h\ p\ n$, the Angles at o and p are right by the Nature of the *Sphere*; therefore the Triangles are Equi-angular; and $o\ b$ being Equal to $h\ p$ as Versed Sines of equal Arches $q\ b$ and $g\ h$, to the common *Radius* of the *Sphere*, therefore $b\ m$ is Equal to $h\ n$, and $o\ m$ to $p\ n$: And $m\ n$ being added on both sides, $b\ n$ is Equal to $h\ m$; and $k\ b$ being equal to $h\ l$, $k\ h\ b = l\ b\ h$; by 27. III. *Euclid*. Therefore in the Triangles $k\ h\ m$ and $l\ b\ n$ the sides $k\ h$ and $l\ b$, $m\ h$ and $n\ b$, with the included Angles $l\ b\ n$ and $k\ h\ m$ being Equal, they are Equal in all respects; and $k\ m = l\ n$; by 4. I. *Euclid*. Therefore in the Triangles $k\ m\ o$ and $l\ n\ p$, $k\ m$ being Equal to $l\ n$, and $m\ o$ to $n\ p$; and $k\ o$ and $l\ p$ being *Radii* of Equal Circles and Consequently Equal, the Triangles are Equal in all respects; by 8. I. *Euclid*; and $k\ o\ q = l\ p\ g$; $k\ q$ to $l\ g$.

Q. E. D.

Corollary 1. From this *Lemma* we learn how to divide a Circle in the Projection, Representative of another Circle in the *Sphere*, as that Circle in the *Sphere* is divided. For (in *Fig* 25.) let the Representative $q\ p\ o$ be divided as the Circle it represents $g\ f\ e$ is divided in k; b being the Place of the Eye projecting $g\ f\ e$ into $q\ p\ o$. 'Tis plain that the Point g is represented by q, and k by p; therefore $q\ p$ represents $g\ k$: And to find out the Point p by the help of a Circle in the Plane of the Projection, a Line drawn from n, the projected Pole of the Circle $g\ f\ e$ which falls within the Plane of the Projection, thro' l, making $a\ l$ in the Primitive Circle Equal to $k\ g$ in the Circle to be projected will fall upon p, and consequently cut the Representative as it is required. For $b\ k$ and $n\ l$ being

being *Inclin'd* to each other, if continued far enough, they will meet, being in the Plane of the Circle *b k h*, somewhere in that Plane continu'd; but *l n* is also in the Plane of the Circle *q p o*, therefore they must meet in the Interfection of those two Planes: But *b k* intersects the Plane *q p o* at *p*; therefore *n l* must meet it at *p*. Therefore a Line drawn from the projected Pole of a Great or Lesser Circle which falls within the Plane of the Projection thro' an assign'd Division of another Circle (*viz.* either of that upon which the Projection is made, if the Circle to be projected is a Great Circle; or of a Lesser Circle parallel to it, if the Circle to be projected is a Lesser Circle equally distant from its Poles as the Circle to be projected is, by the precedent *Scholium*) answering to a Division of the Circle to be projected, will cut the Representative as the Circle it represents is supposed to be cut.

Coroll. 2. Hence it is also easy to discover to what Arch of a Circle an Arch of its Representative in the Plane of the Projection is equivalent: For a Line drawn from *p* to *n* cuts off *a l* = *k g*, which *q p* represents.

LEMMA III.

If a Plane be drawn thro' the Two nearest Poles of Two Equal Circles of the Sphere, *moving on the Line connecting those Two Poles as an Axis; it will cut off Equal Portions of the Circumferences of those Circles, on the contrary side.*

DEMONSTRATION.

Fig. 27. IN the Scheme annex'd, let *b c* connecting the Two nearest Poles *b* and *c* of the Circles *d k e* and

and bkn, be continued 'till it interfects ae and ah continued in the Points f and g; and let the Planes of the two Circles bkn and dke be at Right Angles to the Plane of the Circle $bdne$: In the Triangle bac the Angles at b and c are Equal by *Isosceles* Triangles; and $abc = bga + bag$; and $acb = cfa + caf$; therefore $bga + bag = cfa + caf$; but $caf = bag$, because each of 'em added to bac makes a Right Angle; therefore $bga = cfa$, and $ag = af$. Let us conceive the Plane gaf moving upon the Axis bc continued to pass from a to p; it will become gpf, cutting the Circles assign'd in i and o, l and m if continu'd as far as requisite; and will make $kl = km$, $ld = mn$, $ib = oe$, and ik to ko.

For in the two Triangles gap and fap, the Circles in the Planes of which they are found being at Right Angles to $bdne$, the Angles gap and fap are Equal being Right; and ga and fa being prov'd Equal, and ap being common to both Triangles, they are Equal in all respects, by 4. I *Euclid*; and $gpa = fpa$: Therefore in the Triangles apm and apl, $apm = apl$; being the Complements of two Equal Angles to 180 Degrees; and ap is common to both, and $al = am$, as *Radii* of Equal Circles; therefore (by 7. VI. *Euclid*) $pal = pam$, and $kl = km$; and their Complements to 90 Degrees $ld = mn$. And in the Triangles iap and oap the Angles ipa and opa being already prov'd Equal, and ap being common to both; and $ai = ao$, as *Radii* of Equal Circles; $iak = oak$; $ik = ok$; and their Complements to a Quadrant $ib = oe$.

Q. E. D.

Scholium. This holds also of Equal Lesser Circles as well as great; and this Demonstration proves it if we suppose a to be out of the Centre

of the *Sphere*; for then will the Circles *h k n* and *d k e* which pass thro' the Point *a* be Lesser Circles, equally distant from their Poles *c* and *b*. And the Demonstration will proceed as before; if we suppose *b d n e* an Equal Lesser Circle of the *Sphere* to which they are perpendicular.

Corollary 1. From this *Lemma* we are furnish'd with another way of dividing a Circle in the Projection Representative of another Circle in the *Sphere* as that Circle in the *Sphere* is divided. For if *k r q*, which is evidently the Representative of *k m n*, to the Eye at *b* projecting *h k n* upon the Plane *d k e*, is to be divided as *k m n* is divided in *m*; a Line drawn from *f* the *Projection* of the Pole of the *Inclin'd* Circle nearest to the Eye at *b*, to *l* in the Plane of the Projection making *l d* = *m n*, will cut off *r q* Representative of *m h* = *d l*. For *b m* and *b n* being drawn, 'tis plain that *r q* rrpresents *m n*: If therefore *f l* must necessarilly cut *b m* in *r*, then it must cut off the Representation of *m n*: But *b m* and *f l* being in the same Plane of *g p f*, and Inclin'd to each other, they must meet if continued; and *f l* being in the Plane of *d k e* at the same time; they must meet in a Point where the Two Planes intersect each other; but *b m* cuts the Plane of *d k e* no where but in the Point *r*; therefore *f l* must meet it in the Point *r*: And *f l* must cut off *r q* Representatative of *m n* = *d l*.

Coroll. 2. Hence it is also easy to discover to what Arch of a Circle an Arch of its Representative in the Plane of the Projection is Equivalent: For a Line drawn from *f* to *r*, and continued to *l*, cuts off *d e* = *m n* which is represented by *r q*.

PROP.

PROP. II. Problem.

If a Circle is projected into a Circle, to cut the projected Circle, whether Great or Small, into Parts Representative of any assign'd Divisions of the Primitive.

SOLUTION.

IF the Circle to be projected is *Parallel* to the Plane of the Projection, its Poles will coincide with those of the Circle upon which the Projection is made, in the Centre of the Projection: And if the Circle upon whose Plane the Projection is made is divided as the Circle to be projected, a Ruler laid from the Centre to those Divisions, and continued if requir'd, will cut the Representative into Parts answering to the Divisions in the Circle it represents: As is Evident to every one, without any Necessity a formal Demonstration. If the Circle to be projected is *Inclin'd* to the Plane of the Projection, then its Poles being found out by *Lemma* I. the Two ~~following~~ foregoing *Lemmas* and their *Corollaries* furnish us with Two Ways of dividing its Representative as the Circle itself is divided. And Perpendicular Lesser Circles are divided after the same manner: By *Schol. Lemma* II.

Q. E. F

SECT. IV.

Sect. IV. *Examples to the foregoing Rules, in the* Stereographic *Projection of the* Sphere, *upon several of its Great Circles.*

EXAMPLE I.

The Stereographic *Projection of the* Sphere *upon the Plane of the* Horizon, *to the Latitude of* 51°. 32′ 00″ North.

Fig. 28. IN the Scheme annex'd, let *s æ t q* represent the *Horizon*, upon the Plane of which the *Sphere* is to be projected: Conceive the Eye at *a* the Centre of the Circle, which represents the Two Poles of the *Horizon*, the *Zenith* and the *Nadir*; and the Eye is supposed in the *Nadir*, projecting the Circles of the *Sphere* upon the *Horizontal* Plane.

Draw the two Diameters *s a t* and *æ a q* cutting each other at Right Angles in the Centre of the Projection; they will represent *s a t* the Meridian of the Place, or (which coincides with it) the *North* and *South Azimuth*; *æ a q* the Prime *Vertical*, or *East* and *West Azimuth*; as those Circles pass thro' the Poles of the Projection, are perpendicular to its Plane and cut each other at Right Angles; by *Prop.* I. *Sect.* I. and II. And as all the other *Azimuth* Circles cut each other in the same Poles, they will also be represented by Right Lines passing thro' the Centre of the Projection. To represent therefore any other *Azimuth* Circle in this Projection, we must set off from our *Sector* open to the *Radius* of the Primitive Circle, or that on which the Projectionis made, the Chord of

of its Amplitude from any two of the opposite Cardinal Points; and a Right Line drawn thro' those Two Points found, and the Centre of the Projection, will represent the *Azimuth* Circle requir'd. This is so plain, that it needs no *Example* for Illustration.

All other Circles of the *Sphere*, whether Great or Small, appear Circles also in the Plane of the Projection; by *Prop.* II. *Sect.* I. For the Projection therefore of them, and first, Of Great Circles, beginning with the *Equator*: In the Latitude of 51° 32′ 00″ the *Equator* is *Inclin'd* 38° 28′ 00″ to the *Horizontal* Plane; the Tangent therefore of that *Inclination* set off from *a* to *f*, gives the Centre of the *Equator* *f*; and the Tangent of half its Distance from the Pole opposite to that in which the Eye is plac'd, or the Centre of the Projection where both the Poles of the *Horizon* coincide, *viz.* 25° 46′ 00″, set off from *a* to *e*, gives *f e* the *Radius* of the *Equator*; and consequently *æ e q* as much of that Circle as appears within the Plane of the Projection: All by *Prop.* III. *Sect.* II.

The *North* Pole of the World is distant from the Pole of the *Horizon* opposite to that in which the Eye is plac'd, or the Centre of the Projection, 38° 28′ 00″; therefore by *Sect.* III. *Lemma* I. the Tangent of 19° 14′ 00″, to the *Radius* of the Projection laid off in the Line of Measures from *a* to *c*, gives *a c* the representative of the Distance of the *Zenith* from the *North* Pole of the World, and *c* the Place where that Pole is view'd. Or if the Chord of 38° 28′ 00″ is laid off from *q* to *x*, a Line drawn from *æ* to *x* will cut *a q* in *c* the Pole of the World: For *a c* is the Tangent of $c\,æ\,a = \frac{1}{2}\, x\, a\, q = 19° \; 14′ \; 00″$; by 20 III. *Euclid.*

The Hour-Circle of *Six* Morning and Evening cutting that of *Twelve* represented by *t a s* at

 Right

Right Angles, and consequently passing thro' the Points *æ* and *q* at a Quadrantal Distance from *s* and *t*; and also (as all the other Hour-Circles) thro' the Pole of the World, is *Inclin'd* to the Plane of the Projection, according to the Latitude of the Place; *viz.* 51° 32′ 00″; the Tangent of which set off in the Line of Measures from *a* to *b* gives the Centre of that Circle; which is consequently represented by *æ c q d*. Or, having the Points *æ*, *c*, *q*, the Centre *b* may be found; by 5. IV. *Euclid*: As may also that of the *Equator*, having the Points *æ*, *e*, and *q*.

All the other Hour-Circles, (I mean, those of the *Astronomical* Hours; for of those which take their Rise from the *Horizon*, I shall speak in another Place) cut that of *Six* in the Point *c*, or the Pole of the Globe in Angles of 15, 30, 45, 60, &c. Degrees. Let therefore the Hour-Circle of *Six* be compleated, and draw the Diameters *c d* and *l m* at Right Angles to each other.

The Hour-Circle at *Five* in the Morning and Evening is *Inclin'd* to the Plane of the *Six-a-Clock* Hour-Circle 15 Degrees. The Tangent therefore of that *Inclination*, to the New *Radius* *b c* or *b m*, laid off in the New Line of Measures from *b* to 15, gives 15 the Centre of that Circle, 15 *c* its *Radius*; and consequently *n c o* as much of that Circle as appears within the Projection. All this by *Prop.* III. and *Schol.* 1. *Prop* IV. *Sect.* II.

The Hour-Circle of *Seven* in the Morning and Evening is *Inclin'd* to the Plane of the *Six-a-Clock* Hour-Circle 15 Degrees the contrary Way; the Tangent therefore of those Degrees, to the same *Radius*, set off from *b* to 15 on the contrary Side, gives 15 the Centre of that Circle, 15 *c* its *Radius*, (as this and all other Hour-Circles pass thro' the Poles of the World) and *κ c λ* the Circle it self.

After

After the ſame manner the reſt of the Hour-Circles are laid off, according to their Inclination to the Plane of the *Six-a-Clock* Hour-Circle; their Centres being all found in the Line of Meaſures *l b m* continu'd as far as requir'd on each *Side*: I need not produce more Inſtances; the Matter being ſo plain to thoſe who underſtand the *Propoſition* and *Scholium* now refer'd to, which they are grounded upon.

All Leſſer Circles *Parallel* to the Plane of the Projection, (*viz.* in *Horizontal* Projections the Circles of Altitude) are projected by aſſuming the Tangent of Half their Diſtance from the Pole of the Projection oppoſite to that in which the Eye is plac'd, to the *Radius* of the Projection, for their *Radii*; and that Pole, or the Centre of the Projection, for their Centre; and drawing the Circles themſelves. For all Leſſer Circles have the ſame Poles with the Greater to which they are *Parallel*; by *Coroll. Lemma* I. *Sect.* III. And in the Caſe before us, the Poles of the *Horizon* coincide with the Centre of the Projection; therefore ſo do the Poles of all its Parallels: And their *Radii* are Equal to the Tangents of Half their Diſtances from the Pole of the Projection oppoſite to that in which the Eye is plac'd, to the *Radius* of the Projection; by *Prop.* IV. *Sect.* II.

All Leſſer Circles *Inclin'd* to the Plane of the Projection, are projected by ſetting off from the Centre of the Primitive Circle, in the Line of Meaſures, the Tangents of Half their neareſt and greateſt Diſtances from the Pole oppoſite to that in which the Eye is plac'd; which biſſected, will give the Centres of the Circles to be projected, and their *Radii*, by *Prop.* IV. *Sect.* II.

If therefore the *Tropic* of *Cancer* is to be projected; its leaſt Diſtance from the *Zenith*, or Pole of the *Horizon* oppoſite to that in which the Eye is

 plac'd,

plac'd, is 28° 03′ 00″; (the *Equator* being 51° 32′ 00″ distant from the *Zenith* in the Latitude of *London*, and the *Tropic* of *Cancer* 23° 29′ 00″ less distant than the *Equator*) Half therefore of that nearest Distance is 14° 01′ 30″. Its greatest Distance is 104° 59′ 00″ (consisting of the Distance of the *Zenith* from the *North* Pole 38° 28′ 00″, and the Pole from the *Tropic* of *Cancer*, 66° 31′ 00″) Half that Distance 52° 29′ 30″. The Tangent therefore of 14° 01′ 30″ to the *Radius* of the Projection, set off from *a* to *g*, and that of 52° 29′ 30″ from *a* to *k* give *g k* a projected Diameter of the *Tropic* of *Cancer*, *z* its Centre; and ♋ *g*, ♋ *k* the *Tropic* itself.

After the same manner we may project the *Tropic* of *Capricorn*, *viz.* ♑ *i* ♑; whose nearest Distance from the *Zenith* is 75° 01′ 00″, and its greatest 151° 57′ 00″: And so likewise any Parallel to the *Equator* 'twixt the Two *Tropics* or 'twixt them and either of the Poles. And if a Circle is to be projected, whose nearest and greatest Distance from the *Zenith* are both on one Side of it, for Instance, the *North* Polar-Circle the Tangent also of Half its nearest and greatest Distance from the *Zenith* set off from that Side where the *North* Pole is found, will give its Diameter; and that bissected, its *Radius*.

The Parallels to any other *Inclin'd* Great Circle, are Projected after the same manner with those which are Parallel to the *Equator*. And the Parallels to such Great Circles as stand at Right Angles to the Plane of the Projection, may be drawn by *Sect.* II. *Prop.* II. None of those are of any great Use in *Horizontal* Projections; and therefore I omit any Instances of this kind in this Place.

Now in order to the Mensuration of any Projected Great or Lesser Circle; *First*, If they are projected

projected in *Right Lines*, we are taught how to measure 'em, by *Prop*. I. *Sect*. III. Thus, supposing I am to cut off 60 Degrees 'twixt *a* and *q*, from *æ a q* the Representative of the *Prime Vertical*, the Tangent of 30 Degrees to the *Radius* of the Projection, laid off from *a* to *w*, gives *a w* in the Circle projected, answering to 60 Degrees in that which it represents. Or if the Chord of 60 Degrees is laid off from *s* to *y*, a Line drawn from *t* to *y*, will determine the Point *w*; and consequently will give us *a w* the Division requir'd; by 20. III. *Euclid*.

Secondly, If they are projected in *Circles*, we have Two ways furnish'd us for the Mensuration of them, whether Great or Small, by the *Corollaries* to the Two latter *Lemmas* in the preceding *Section*. For, *First*, if the Representative of the *Equator* *æ e q* is to be divided according to any assign'd Division of the *Equator* itself; Lines drawn from the Pole of the *Equator* which falls within the Plane of the Projection *c*, to like Divisions in the *Horizon s æ t q*, will cut *æ e q* as the *Equator* itself is cut. Thus if the Representative of 45 Degrees in the *Equator* is to be taken off 'twixt *æ* and *e*, the Chord of 45 Degrees being laid off from *e* to ♉, a Ruler from *c* to ♉ will cut off *æ* ♌ Representative of 45 Degrees in the *Equator*; by *Coroll*. 1. *Lemma* II. *Sect*. III.

If a Lesser Circle is to be divided, for Instance, the Projection of the *Tropic* of *Cancer* by *Schol*. *Lemma* II. and *Coroll*. 1. It may easily be done: For Projecting a Lesser Circle Parallel to the Plane of the *Horizon*, equally distant from its Poles, as the *Tropic* of *Cancer* is from its; Lines from the Representative of the *North* Pole of the Globe to any assignable Divisions of that Circle, will cut the *Tropic* of *Cancer* in the Projection accordingly as you would have it divided. Thus the *Tropic*

Tropic of *Cancer* is distant from the *North* Pole of the Globe 66° 31′ 00″; a Circle therefore must be projected equally distant from the *Nadir*, or farthest Pole of the *Horizon* from the *North* Pole of the Globe; which will be 113° 29′ 00″ from the *Zenith*; the Tangent of Half which, *viz.* 56° 44′ 30″ set off from *a* to ♊, gives *a* ♊ its *Radius*, by *Sect.* II *Prop.* IV. and the Circle it self ♈ ♍ ♎ ♊. If we wou'd therefore cut off the Representative of 30 Degrees in the *Tropic* of *Cancer*, beginning from *g* on the Side *g w* ♋; the Chord of 30 Degrees, to the *Radius a* ♊, in the Circle ♈ ♍ ♎ ♊, laid off from the Point ♊, reaches to ♐; upon which, and the Point *c*, a Ruler being laid, cuts the projected *Tropics* of *Cancer* in the Point ♒; and consequently *g* ♒ is the Representative of 30 Degrees in the *Tropic* of *Cancer*. Whoever throughly understands the *Second Lemma* of the preceding *Section*, with its subsequent *Scholium*, and *First Corollary*, must, without much difficulty, see the Demonstration of this; which therefore stands in need of no further particular Explication.

Secondly, By *Lemma* III. and its *First Corollary*, we are furnished with Rules for Dividing Great or Less Circles of the *Sphere*, by Lines drawn from those their Poles which fall not within the Projection: But of these Methods the Reader must, by this time, be able to give Instances himself.

EXAMPLE II.

The Stereographic *Projection of the* Sphere *upon the Plane of the* Ecliptic.

Fig. 29. IN the Scheme annex'd, let *a æ b μ* represent the *Ecliptic* upon the Plane of which the *Sphere* is to be projected by the Eye, in its Under Pole, represented by *c*. Draw the Lines *b c a* and *æ c μ* at

at Right Angles to each other: And let *b c a* represent the *Solstitial*, and *a c μ* the *Equinoctial Colure*; which both passing through the Poles of the *Ecliptic*, must be represented by Right Lines; by *Sect.* II. *Prop.* I. As will also all the Circles of Celestial Longitude, for the same reason; which may consequently easily be drawn, after the same manner as the *Azimuth* Circles are in *Horizontal* Projections.

The *Equator* is *Inclin'd* to the Plane of the Projection 23° 29′ 00″; the Tangent therefore of that Angle to the *Radius* of the Projection set off from *c* to *h*, gives *h* its Centre; and it is distant from the Pole of the Projection opposite to that in which the Eye is plac'd, 66° 31′ 00″; the Tangent therefore of Half that Angle, *viz.* 33° 15′ 30″, set off from *c* to *q*, gives *h q* the *Radius* of the *Equator*, and consequently *a q μ* the *Equator* itself; by *Sect.* II. *Prop.* III.

The Poles of the *Equator* and the *Ecliptic* are distant 23° 29′ 00″; the Tangent therefore of 11° 44′ 30″ set off from *c* to *p*, gives *p* the Place where the *North* Pole of the World is viewed, by *Sect.* III. *Lemma* I.

As the Circles of Celestial Longitude cut each other in the Poles of the *Ecliptic*, and consequently in the Centre of this Projection, and are projected in Right Lines; so the Circles of Terrestrial Longitude, or Celestial Right Ascension, cut each other in the Poles of the World, and consequently are *Inclin'd* to the Plane of the Projection, and projected in Circle; by *Sect* I. *Theor.* II. And as *a c μ* represents the *Equinoctial Colure* from whence the Degrees of Celestial Longitude are reckon'd in the *Ecliptic*, beginning (for Instance) at the Point *μ*, as the first Point of ♈; so the Degrees of *Celestial* Right Ascension are reckon'd from the same Point in the *Equator*; or by Angles

Angles form'd at the Pole of the World with *æ p μ*; which is *Inclin'd* to the Plane of the Projection 66° 31′ 00″. The Tangent of which laid off from *c* to *m*, gives *m p* the *Radius* of that Circle; and consequently *æ p μ* as much of it as appears within the Projection.

All the other Circles of Celestial Right Ascension, or Terrestrial Longitude, cut the Circle *æ p μ* in the Point *p*; which, if it is completed, they may be drawn according to their different *Inclinations* to its Plane; as the rest of the *Hour-Circles* are upon the Plane of *c l d m* in the *Horizontal* Projection. I have drawn but Six of 'em here *Inclin'd*; Two 15, Two 30, and Two 45 Degrees to the Plane of *æ p μ*, suppos'd to be completed.

All Lesser Circles *Parallel* to the Plane of the Projection; *viz.* the Circles of Celestial Latitude, are projected as the Circles of Altitude in the *Horizontal* Projection: And consequently may easily be drawn, when there is occasion for 'em, in the Solution of *Astronomical* Problems. But no Instances need be given of them in this Place.

Lesser Circles *Inclin'd* to the Plane of the Projection, (for Instance, those Parallel to the Plane of the *Equator*) are projected, by setting off from the Centre of the Projection, in the Line of Measures, *b c a*, the Tangents of Half their nearest and greatest Distances from the Pole of the Projection opposite to that in which the Eye is plac'd, which coincides with the Centre of the Projection; which bissected, will give the Centre of the Circle to be projected, and its *Radius*; by *Sect.* II. *Prop.* IV. Thus in the Projection of the *Tropic* of *Cancer*, its least Distance from the Upper Pole of the *Ecliptic* is 43° 20′0 0″; the Tangent of Half which laid off from *c* to ♋, gives ♋ for one End of its Projected Diameter; its greatest Distance is 90 Degres, the Tangent

Tangent of Half of which is Equal to the *Radius* of the Projection *c b*; therefore ♋ *b* isthe Projected Diameter of this *Tropic*; which bissected, gives *i* its Centre; and ♋ *u b* ß the *Tropic* it self.

After the same manner are projected all Parallels to the *Equator*, and to any other Great Circle, however *Inclin'd* to the Plane of the Projection: Only if it cuts the Plane of the Projection in Points different from *æ* and *μ*, the Line of Measures will be at Right Angles to a Line drawn to the New Points of Intersection, as *b c a* is at Right Angles to *æ c μ*, upon which New Line of Measures the Diameters of its Parallels are to be found.

If the *Earth* turns upon its own Axis, or the Axis of the *Equator*, in 24 Hours, then the *Vertex* of every Place upon it describes a Circle in that Time Parallel to the *Equator*, which may not improperly be call'd, *The Path of the Vertex of the Place*; which Circle is the same with the Parallel of its Latitude. The Parallel of Latitude therefore of any Place being projected at the Path of its *Vertex* is projected at the same time: And the Parallels of Latitude are projected after the same manner as the *Tropic* of *Cancer*.

Thus if the Parallel of Latitude or Path of the *Vertex*, of *London* is to be projected; its Latitude being 51° 32′ 00″ *North*; its Distance from the Pole of the Globe must be 38° 28′ 00″. Therefore its Distance from the Upper Pole of the *Ecliptick*, 14° 59′ 00″, the Distance of the Two Poles of the *Ecliptick* and *Equator* 23° 29′ 00″ being deducted from 38° 28′ 00″ the Tangent of Half which, 7° 29′ 30″, laid off from *c* to *L*, gives *L* one of the projected Extremities of the Diameter of the Parallel or Path of the *Vertex*, of *London*; and its greatest Distance from the same

ſame Pole is 61° 57′ 00″; the Diſtance of the Two Poles, and of itſelf, from the Pole of the World, being added together; the Tangent of Half which 30° 58′ 30″ laid off from *c* to *k*, gives *L k* the Diameter of that Path; by which biſſected, we find *L π k v* to be *the Projected Path of the* Vertex *of* London.

The Earl of *Sandwich* obſerv'd by the Sun's *Solſtitial* and other *Meridional* Altitudes, that the Latitude of *Madrid* is 40° 10′ 00″ *North*. Therefore its Diſtance from the Pole of the Globe is 49° 50′ 00″: From which taking away 23° 29′ 00″, the Diſtance of the Poles of the *Equator* and *Ecliptic*, its neareſt Diſtance from the Pole of the *Ecliptic*, or Centre of the preſent Projection, is 26° 21′ 00″; and adding 23° 29′ 00″ to 49° 50′ 00″, its greateſt Diſtance is 72° 19′ 00″. The Tangents of Half which Diſtances ſet off, the former from *c* to *M*, and the latter from *c* to *l*, give *l M* the Projected Diameter of the *Path of the Vertex* of *Madrid*; and *M δ l φ the Projected Path of its Vertex.*

Perpendicular Leſſer Circles are of no more Uſe in this than in the Precedent Projection: And all Circles are divided here, as in the former Caſe. Thus if we wou'd cut the Projected *Equator* in any aſſignable manner, a Line drawn from its Pole within the Projection to a like Diviſion in the *Ecliptic* does the thing. And the *Tropic* of *Cancer* is divided here, as in the former Caſe. And after the ſame manner may any other Parallels of Latitude, or *Paths of Vertices* be divided. As every one muſt ſee, without the help of Inſtances for Illuſtration.

EXAMPLE

Example III.

The Stereographic *Projection of the* Sphere *upon the Plane of the* Equinoctial Colure.

IN this Projection the Eye is suppos'd in the 90th Degree of Right Ascension from the First Point of *Libra* in the Plane of the *Equator*, and in one of the Poles of the *Equinoctial Colure*. Therefore the *Equator* in this Place is represented by the Right Line ♈ *a* ♎; *p a* ☾ at Right Angles to it representing the Axis of the World. And the *Equator* being perpendicular to the Plane of the Projection, the *Ecliptic* is *Inclin'd* to it in an Angle of 66° 31′ 00″. The Tangent of which set off from the Centre of the Projection from *a* to *a* gives *a* the Centre of the *Ecliptic*: And the *Ecliptic* being 23° 29′ 00″ distant from the Pole of the Projection opposite to that in which the Eye is plac'd, the Tangent of 11° 44′ 30″ set off from *a* to *n*, gives *a n* its *Radius*; and consequently ♈ *n* ♎ the *Ecliptic* it self. By *Sect.* II. *Prop.* III. *Parallels* to the *Ecliptic* are *Inclin'd* to the Plane of the Projection; and consequently are projected as Parallels to the *Equator* in the Projection upon the Plane of the *Ecliptic*. Fig. 30.

Parallels to the *Equator* being perpendicular to the Plane of the Projection are projected according to the Direction of *Prop.* II. *Sect.* II. Thus the *Tropic* of *Capricorn*, having its Poles in the Plane of the Projection, is distant from the *South* Pole of the World 66° 31′ 00°; the Secant therefore of that Distance set off from *a* to *b*, gives *b* the Centre of that *Tropic*, and the Tangent of the same *b* ♑ its *Radius*; and consequently ♑ *m* ♑ is the *Tropic* it self. And if you are not provided with a Line of Secants already, 'tis easy to discover the

the Secant you want to the *Radius* of your Projection in the Scheme itself: For laying off the Tangent of 66° 31′ 00″ from *æ* to *a*, and drawing the Line ♎ *a*, it is the Secant of that Arch and easily transferable to *æ b*. Or the Chord of 66° 31′ 00″, to the *Radius* of the Projection, being laid off on both sides the Point *π*, gives Two Points ♑ ♑ in the Limb of the Projection thro' which this *Tropic* must pass; and as it cuts the *Solstitial Colure*, represented here by *p æ π* at 23°. 29′ 00″ Distance from the *Equator*, the Tangent of half that Distance laid off from *æ* to *m* represents that Arch, and gives *m* the Point of Intersection; by *Sect.* III. *Prop.* I. thro' which and the Two Points ♑ ♑ a Circle drawn by 5. IV. *Euclid*, gives us that *Tropic*. After the same manner are drawn the *Tropic* of *Cancer*, *viz.* ♋ *n* ♋, the Parallel of *London L i l*, and all other Parallels to the *Equator*.

The Hour-Circles, or Circles of Terrestrial Longitude, or Celestial Right Ascension, cutting each other in the Poles *p* and *π*, are easily projected by setting of the Tangents of their respective *Inclinations* to the Plane of the Projection in the Line of Measures ♎ *æ* ♈ continued as far as wanted; and the Tangents of half their Distances from the Pole of the Projection opposite to that in which the Eye is plac'd the contrary Way, in order to have their *Radii*, and then the Circles themselves: By *Sect.* II. *Prop.* III. as every one must see by looking upon the Figure, without requiring any particular Explication of the Matter.

If any of the great Circles projected here into Right Lines are to be divided, the same Method must be taken as is observ'd in the Division of the *Prime Vertical* in the *Horizontal* Projection. And those Lesser Circles which are *parallel* to the Great ones

which are projected into Right Lines, or *perpendicular* to the Plane of the Projection. For Instance, the *Tropics* and their Parallels of Terrestrial Latitude or Celestial Declination may be divided after the same manner with the *Tropic* of *Cancer* in the *Horizontal* Projection: Thus if you wou'd divide the Projected *Tropic* of *Cancer* at pleasure, or a Quadrant of it, into Three Parts Representative of 30, 60, and 90 Degrees; its Distance from the *Northern* Pole of the World is $66° \; 31' \; 00''$; and a Circle equally distant from the Pole of the *Equinoctial Colure* in which the Eye is plac'd, is distant from the opposite Pole $113° \; 29' \; 00''$; the Tangent therefore of $56° \; 44' \; 30''$ set off from *a* to ♌ will give us the *Radius* of that Circle, by *Prop.* IV. *Sect.* II. and ♌ *E* ♉ *F* the Circle it self: The Quadrant of which *F* ♌ having the Chords of ♌ 30, ♌ 60, laid off to the *Radius* *a* ♌, Lines drawn from *p* to 30 and 60, will cut the *Tropic* of *Cancer* as requir'd in *n* Ψ, by *Schol.* and *Coroll.* I. *Lemma* II. *Sect.* III.

Or all the Parallels to the *Equator*, and in particular the *Tropic* of *Cancer*, may be thus divided: Let the Tangent of 30 Degrees to the *Radius* of the Projection be laid off from *a* to φ; *a* φ will represent 60 Degrees in the *Equator*, by *Sect.* III. *Prop.* I. And a Circle struck thro *p* φ π will cut the *Tropic* of *Cancer* in Ψ, so that *n* Ψ shall represent 60 Degrees in the *Tropic* of *Cancer*. For the Circles represented by *p* φ π and *p* *a* π being *Inclin'd* to each other in an Angle of 60 Degrees, and at Right Angles to the *Equator*; their Planes intersecting each other in the Axis of the *Sphere*, must cut off Similar Arches from all the Parallels to the *Equator*, from each Parallel 60 Degrees; and the Representatives of those Arches being necessarily included in the Projection 'twixt the Representatives of the Circles which determine 'em, they

will be cut accordingly: And therefore whatever Arch you take off from the Repreſentative of the *Equator*; a Circle drawn through the Point that determines it, and the Two Poles of the Globe, will cut the Repreſentatives of all the Parallels to the *Equator*, after the ſame manner.

If the *Ecliptic* is to be divided into its Signs, its Pole, which falls within the Plane of the Projection, is diſtant from the Centre of the Projection the Tangent of Half its *Inclination* to its Plane; by *Sect.* III. *Lemma* I. But the *Ecliptic* is *Inclin'd* to the Plane of the Projection 66° 31′ 00″; The Tangent therefore of 33° 15′ 30″ ſet off from *a* to *n*, gives that Pole: From which Lines drawn to *α*, *β*, *γ*, *δ*, (♈ *α*, *α β*, &c. containing 30 Degrees each) will cut the *Ecliptic* in ♉, ♊, *&c.* where thoſe Signs begin; by *Sect.* III. *Coroll.* 1. *Lemma* II. Parallels to the *Ecliptic* are divided here, as Parellels to the *Equator* in the *Horizontal* Projection. And thus all other *Inclin'd* Circles, as the Hour-Circles, and others are divided; as they may alſo be by *Coroll.* 1. *Lemma* III. *Sect.* III. and by ſeveral other Ways to be met with in the Writers upon *Stereographick Projections*; which we have no occaſion to take particular Notice of in this Place. And having an Arch of a Repreſentative, if we deſire to know its Value in the Circle it repreſents, it is eaſily done by the *Second Corollary* of either the *Second* or *Third Lemma.* Thus (*Fig.* 28.) If we have a mind to know how much of the *Equator* is repreſented by *a* ♌ a Ruler laid upon the Pole *c*, and the Point ♌ *e*, will cut the *Horizon* in ♉, giving us *a* ♉ equal to that part of the *Equator* which *a* ♌ repreſents; which will be found upon the Line of Chords of the *Sector*, open to the *Radius* of the Projection, to be 30 Degrees. If we have a mind to know how much of the *Tropic* of *Cancer* is repreſented by

g ♒,

g ♒, a Ruler laid upon the Pole *c* and the Point ♒, will cut the Circle ♈ ♍ ♎ ♊ in the Point ♐, giving us ♊ ♐ Equal to that part of the *Tropic* of *Cancer* which *g* ♒ represents; which will be found upon the Line of Chords upon the *Sector*, open to the *Radius a* ♊, to be 30 Degrees. But these being but the Converse of *Prob.* II. *Sect.* III. scarce need to have been named in this Place. The Converse of *Prob.* I. teaches us, having any Portion of a Right Line Representative of a Great Circle, of what Value that Portion is in the Circle itself.

And thus much, I conceive, may be fully sufficient to give the Reader a just Notion of the whole Business of *Orthographic* and *Stereographic* Projections; and to enable him to project the *Sphere* upon the Plane of any other Great Circle, as well as those I have Instanc'd in. The Planes I have chosen, are the most commonly made use of; and will be the most to my Purpose, when I come to shew the Use of Projections in the Solution of *Astronomical* and *Geographical Problems*: Which I shall do in the Sequel of this Discourse; beginning with the Use of the *Orthographic* and *Stereographic* Projection of the *Sphere in Plano*, in the Solution of *Astronomical* Problems.

G 2 CHAP.

CHAP. III.

The Use of the Projection of the SPHERE in Plano, *in the Solution of* Aſtronomical Problems.

EVery one who has been let into the *Firſt* Rudiments of *Aſtronomy* knows, that it conſiſts of Two General Parts, the *Spherical* and the *Theorical*: The *Spherical* Part is concern'd principally in explaining the Origin, Cauſes, and Diviſions of thoſe Circles of the *Sphere* which are made uſe of in the Solution of thoſe *Appearances* in the Heavens which reſult from the Diurnal Motion of the *Earth* upon its own *Axis*, and in the Actual Solution of ſuch Problems as thoſe *Appearances* furniſh us with, by the Help of thoſe Circles of the *Sphere*. And the *Theorical* Part is primarily converſant about the Determination of the Orbits of the Planets and their proper Motions, from the Obſervations relating to them, made from time to time by induſtrious and accurate Men, in order to enable us to frame Tables from which to predict, or to determine to any time paſt ſince the Foundation of the World, their Places, Poſitions, Conjunctions, Oppoſitions, Aſpects, &c. The latter of theſe the Projection of the *Sphere in Plano* has little or no relation to; but it is not only uſeful, but neceſſary to the Solution of all the Problems reſulting from the Diurnal *Phænomena*; to which therefore I ſhall confine

confine myself in this Chapter: Assuming as Fundamentals to *Astronomy* in general the following *Postulata*.

Postulat. I. *That the* Earth *turns round upon its own* Axis *once in* 24 *Hours; and that this Diurnal Motion is always Equable, Uniform, and the same, in every Point of its Orbit.*

Postulat. II. *That besides its* Diurnal *Motion upon the* Axis *of the* Equator; *it has an* Annual *Motion about the Sun, nearly in the Centre of its Orbit, having its own Centre always in the Plane of the* Ecliptic.

Schol. These Two *Postulata* are what have been admitted by all the most Impartial and Judicious *Astronomers*, since *Copernicus*'s Time; and they, with almost all the present System of *Astronomy*, were asserted long before the Commencement of the *Ptolemaic* System. And indeed whoever reads what has been advanc'd in Defense of this Old *Pythagorean* System of the World by *Galileo*, *Keppler*, *Bullialdus*, *Gassendi*, and other Famous Masters in *Astronomy* then, and since that time, must admit these Particulars to have almost absolute Demonstration on their side; at least to be infinitely more probable than *Ptolemy*'s Extravagant Diurnal Motion of the *Eighth Sphere*, and his Bungling Fardle of *Epicycles* invented to account for the Stations, Retrogradations, *&c.* of the Planets; which are infinitely more naturally accounted for by the Translation of the *Earth*'s Centre in the Space of a Year, in the Plane of the *Ecliptic* round the *Sun*, which is near the Centre of the *Earth*'s Orbit, but strictly in one of the *Foci* of the *Ellipsis* which it moves in.

Postulat. III. *That the* Axis *of the* Earth, *during its whole Translation thro' its* Annual *Orbit, keeps the same* Inclination *to the Plane of the* Ecliptic; *and is in all Points of its Orbit Parallel to itself.*

Postulat. IV. *That the whole Diameter of the* Orbis Magnus, *or the* Earth's *Annual Orbit, is but a Point in Comparison of the Inconceivably Immense Distance of the* Fix'd Stars.

Schol. Tho' these Two Latter *Postulata* are not strictly and Geometrically true, yet they are so very near it that their Truth may be very safely taken for granted: The small Variation in the *Inclination* of the Planes of the *Equator* and *Ecliptic*, in different Periods of Time being altogether Insensible, and only Discoverable by Deductions from *Physical* Reasonings *à Priori*; and the Parallax which the Diameter of the *Earth*'s Annual Orbit makes with some of the *Circum-Polar* Fix'd Stars (tho' it be a good Argument to establish the Annual Motion of the Earth) is so small, that to the Eye in those Stars it may be considered as Intirely Insensible.

Coroll. I. From the *Three* former of these *Postulata* we may deduce the Reasons of the Variations of the Seasons in all the Parts of the Earth, at different times of the Year, resulting from the different *Meridional* Heighths of the *Sun*, and Lengths of the Days and Nights, wherever there

Fig. 31. is any Variation in them. For in the Scheme annex'd, Let ♉ ♌ ♏ ♒ represent the *Earth*'s Annual Orbit, having ☉ (the *Sun*) nearly in its Centre; which I have made absolutely so here, because the Supposition of its being in one of the *Foci* of that *Ellipsis* which the *Earth* really moves in, is of no

Con-

Consideration to my present purpose: Thro' the Centre of this Orbit draw *c* ⊙ *æ*; representing the *Equinoctial*, and *a* ⊙ *g* the *Solstitial* Colure; and upon the *Four* Cardinal Points where they Intersect the *Earth's* Orbit, mark'd all here with *e*, project the *Sphere Stereographically* upon the Plane of the *Ecliptic*, according to the Method observ'd in the *Second* Example in the *Fourth* Section of the preceding Chapter: 'Tis plain that *c æ* is the *Equinoctial* Colure, or more properly, the Representative of the Intersection of its Plane with the Plane of the Projection, and *a g* the *Solstitial*, in all these Projections; *p* the Pole of the *Globe*; *æ F c* the *Equator*; *μ d b a* the *Tropic* of *Cancer*; *l m L h* the *Path* of the *Vertex* of *London*; and the Circles which cut each other in *p*, the Pole of the World, are Circles of Right Ascension in the *Celestial*, and of Longitude in the *Terrestrial Sphere*; as in the Example referr'd to. And as the *Sun* only illuminates one half of the *Earth's* Globe at the same time, the Line which divides the Illuminate from the Obscure Part of the *Earth's* Disc, may not improperly be call'd *the Horizon of the Earth's Disc*, as *æ e c*, *æ e c* in the *Tropics* of *Cancer* and *Capricorn*, and *a e g*, *a e g* in the *Equinoctial* Points of ♈ and ♎: And the Plane of the Projection infinitely continued every way, will at last coincide with the *Ecliptic* amongst the Fix'd Stars, or that Orbit which the Centre of the *Earth* in its Annual Revolution seems to describe amongst 'em to an Eye in the *Sun*, which is therefore call'd *the Plane of the Ecliptic in the Earth's Disc*, or (when there is no Occasion to distinguish it from the true *Ecliptic*) simply *the Plane of the Ecliptic*, or *the Ecliptic* itself. Now the Axis of the *Earth* in this Projection being apparently in the same Plane with the *Solstitial* Colure, and that passing thro' the Eye in the *Southern* Pole of the *Ecliptic* where the Eye

is plac'd in this Projection, and therefore being projected into a Right Line, that Right Line must represent both at the same time, and consequently *p e g* must represent so much of the *Earth's* Axis as falls within these Projections: And the *Earth* turning upon this Axis to the *Sun* once in 24 Hours, by *Postulat.* I. and in its Translation thro' its Annual Orbit (assum'd in *Postulat.* II.) keeping its Axis always equally *Inclin'd* to the Plane of the *Ecliptic*, and therein *p e* the Tangent of Half the *Inclination* of the Planes of the *Equator* and *Ecliptic* constantly the same, and the same Axis, or the Line *p e g* (which Line for that reason is not improperly call'd *the Line of Direction of the Earth's Axis)* constantly Parallel to itself; by *Postulat.* III. From hence we may easily determine the Reasons of the forementioned *Phænomena* by mere Inspection into the present Scheme.

For let the *Earth* be in the *Tropic* of *Capricorn* viewing the *Sun* in the Opposite Point of the *Ecliptic*, *viz.* the *Tropic* of *Cancer*, and let the *Vertex* of *London* (for Instance) be conceiv'd to move in its *Path* from *m* its Place at Midnight, thro' *l* the Place at Sun-rising, to *h* its Place at Noon, and so on to *L* its Place at Sun-setting, till it comes in 24 Hours time to *m* again. 'Tis plain that then the Illuminate Part of the *Earth's* Disc is *a a c*; and consequently, whereas the *Vertex* of *London* moves thro' the whole *Path e h L m* in 24 Hours, the Illuminate or Diurnal Arch of its *Path l h L* is much larger than the Obscure or Nocturnal Arch; and consequently the Day, when the *Earth* is in the First Point of *Capricorn*, and the *Sun* in that of *Cancer*, must be much longer than the Night. And if we project the *Earth* in ever so many Points in the Quadrants of its Annual Orbit *e* ♏ ♐ *e*, and *e* ♒ ♓ *e*, we shall always find the Diurnal longer than the Nocturnal Arches of the *Path*, till we come to the

First

First Points of ♈ and ♎, decreasing on each Side from *e* in the *Tropic* of *Capricorn*, where we have the Longest Day, the *Sun* being then view'd in the *First* Point of *Cancer*, and having his Central Rays running Parallel to the great *Solstitial* Colure. The same holds of the *Paths* of the *Vertices*, or Parallels of Latitude of all other Places 'twixt the *Equator* and the *North* Pole of the Globe; and the contrary in the Opposite Hemisphere from the *Equator*, to the *South* Pole; as any one may see who will project that Part of the *Sphere* upon the Plane assign'd, and whereas in the Projection of the *Equator*, as in all other great Circles of the *Sphere*, half of it is always above the Plane of the Projection and half below; half of it in the Illuminate and half of it in the Obscure Part of the *Disc*; hence it follows, that under the *Line* the Days and Nights are equal all the Year round: And in every Parallel of Latitude 'twixt the *Equator* and the *North* Pole the Days are longer in Proportion to their Nearness to the Pole, and the contrary on the other Side of the *Equator*; thus in the Case of the Parallel of *London*, the Nocturnal Arch *l m L* bears a less Proportion to the whole *Path h l m L*, than in the *Tropic* of *Cancer* or the Parallel of 23° 29″ 00′, the Nocturnal Arch *u d b*, does to the whole *Path a u d b*: For whereas the Hour-Circles in this Projection are two Hours distant from each other, and *a p g* represents that of *Noon* and *Midnight* to all Places under the same *Meridian* with the City of *London*; 'tis plain that the Nocturnal Arch of its *Path* does not take up quite *Eight* Hours, whereas that of the Parallel of 23° 29′ 00″ takes up more than *Ten*, the shortest Night in our Latitude upon Caculation appertaining to be only 07ʰ 34′ 30″, whereas that under the Latitude of 23° 29′ 00″ is no less than 10ʰ 10′ 56″. And so the Length of the

the Day increases 'twixt us and the *North* Pole, till at last it is 24 Hours in that *Path* which just touches upon the Horizon of the Disc, and no less than half a Year to those who live actually under the *North* Pole of the World. At this Time also the *Sun* is the nearest our *Vertex* it can come, it being in that Point of the *Ecliptic* *a* in which it touches the *Tropic* of *Cancer*, and therefore but 28° 03′ 00″ from our *Vertex*.

Let the *Earth* move along in its Orbit till it comes to the first Point of *Aries*, and it will view the *Sun* in the *First* Point of *Libra*; and as it keeps its Line of Direction *p e g*, or *p e*, constantly Parallel to itself, the *Horizon* of the *Disc* in the *First* Point of *Aries* will stand at Right-Angles to that in the *First* Point of *Capricorn*; *viz.* *a e g*; which is the same with the Line of Measures in which the Centres of all the Paths of the *Vertices* are found; and consequently more or less of that will be a Diameter to each of 'em; and a Diameter cutting every Circle into Two Equal Parts, the *Path* of each *Vertex* will be half in the Illuminate and half in the Obscure Part of the *Earth*'s Disc; and consequently the Days and Nights will be Equal to all the Inhabitants of the *Earth*: and the direct Central Rays of the *Sun* running Parallel to the Great *Equinoctial* Colure, and cutting the *Earth*'s *Ecliptic* in one of those Points where the *Equator* and *Ecliptic* intersect each other, the *Sun* will be distant from the *Vertex* of *London* 51° 32′ 00″, the same as the Distance of the *Equator* from that *Vertex* or the Latitude of the Place. The same happens when the *Earth* is in the *First* Point of *Libra*, viewing the *Sun* in the *First* Point of *Aries*; for the same Reasons.

And when the *Earth* in its Annual Motion comes to the First Point of *Cancer*, viewing the *Sun* in the *First* Point of *Capricorn*; its Line of Direction

rection *p e* ſtill keeping Parallel to itſelf, the Nocturnal Arches of the *Paths* of the *Vertices* are Equal to the Diurnal when the *Earth* is in the *Tropic* of *Capricorn*, and increaſe each Path according to its Greater *Northern* Latitude. And the Central Rays from the *Sun* falling then upon that Point of the *Ecliptic g*, where it is moſt remote from the upper part of the *Equator*, his Diſtance from our *Vertex* will then be the greateſt; *viz*. 75° 01′ 00″. And in all the intermediate Points 'twixt ♈ and ♎ the Nights will be Longer or Shorter in Proportion to the *Earth*'s Nearneſs to or Remoteneſs from the *Firſt* Point of *Cancer*; as every one may ſee who'll take the Pains to Project the *Sphere* of the *Earth* in any one of thoſe Points upon the Plane of the *Ecliptic*.

Coroll. 2. From the *Fourth Poſtulatum* it follows, that in ſolving the Diurnal *Phænomena*, we have no occaſion to conſider the *Earth* as in the Circumference of its Annual Orbit, but in the Centre of it; ſince the Diſtance of its Centre from the remoteſt Parts of its Circumference is but an inſenſible Point in Compariſon of the Immenſe Diſtance of the Fix'd Stars. And whereas, to an Eye in the *Sun*, the *Earth* does really appear to move round the *Sun* in that Circle which we call the *Ecliptic* amongſt the fixed Stars once in a Year; to an Eye in the *Earth*, conceiving itſelf as Quieſcent, the *Sun* muſt ſeem Annually to move in the ſame Orbit round the *Earth*: And conſequently may be conceived at any time to be in that Point of the *Ecliptic* in the *Earth*'s *Diſc* upon which its Central Rays fall perpendicularly at that time: For Inſtance, when the *Earth* is in the *Firſt* Point of *Capricorn*, the *Sun*'s Central Rays falling then upon *a* the *Firſt* Point of *Cancer* in the *Earth*'s *Diſc*, the *Sun* may be ſaid then to be in the *Firſt* Point of *Cancer* in that *Diſc*; as it is actually then

then view'd in that Point in the Heavens: And so for all the other Points of its Orbit. These things being well consider'd and admitted, it will not be difficult to solve all the most Momentous *Astronomical* Problems relating to the *First* Motion upon this Projection; or any other, either, *Orthographic* or *Stereographic*, which we are willing to pitch upon. I shall make use of a large *Stereographick* Projection of the *Sphere* upon the Plane of the *Ecliptic* (*Fig.* 32.) at this time, in which I will suppose the *Sun* in ♉ 12° 15′ 13″; where it will be found to be in 1715. *Apr.* 21^{d} 21^{h} 46′ 33″; or on the 22d of *April*, at 09$_{h}$ 46′ 33″ in the *Morning* to the *Meridian* of *Greenwich*. And from this one Instance the Intelligent Reader will easily learn how to solve all the Diurnal *Phænomena* upon this Projection to any Day in a Year, having the *Sun*'s Place then in the *Ecliptic* from the best *Solar Tables* he can meet with. And he will see how to do the same upon any other Plane, if he is throughly acquainted with the Two preceding Chapters, and with the Natures and *Astronomical* Uses of the Principal Circles of the *Sphere*; which I have already taken for granted as previous to the Full Understanding of this Book.

Of the Diurnal *Phænomena* some are more *General*, being the same to all the Inabitants of the *Earth* at the same time; and others are more *Particular*, requiring our Knowledge of the Latitude of the Place we are in before we can determine 'em: The Solution of such Problems as relate to each of these Kinds of Appearances shall be the Business of the following Part of this Chapter: And I shall begin with those which are more *General*, and are the same to all the Inhabitants of the *Earth* at the same time.

SECT.

SECT. I.

The Solution of such Problems resulting from the Diurnal Phænomena, *as are the same to all the Inhabitants of the* Earth *at the same time.* Sect. I.

PROBLEM I.

Having the Sun's *Place in the* Ecliptic, *and the Inclination of the Planes of the* Equator *and* Ecliptic *to each other; to find out his* Right Ascension *from the* First Point *of* Aries, *his Distance from the* North *Pole of the World, and the Angle which the* Meridian *passing thro' the* Sun *in that Place makes with the* Ecliptic.

SOLUTION *upon the Projection.*

IN the Scheme annex'd let ☉ represent the *Sun's* Place in the *Ecliptic*, which I here assume to be in ♉ 12° 15′ 13″, the Arch ♈ ☉ being 42° 15′ 13″. 'Tis plain that *f* ☉ *g* is the Illuminate, and *f d g* the Obscure Part of the *Earth's Disc*; and if we continue *f e g* the *Horizon* of the *Disc* till it meets with *a b c* the Line of Measures in which the Centers of all the Hour-Circles, or *Meridians*, are found, it will cut it in *c*, the Centre of the *Proper Meridian*, to that Place of the *Sun* *d p* ☉, *c* ☉ being its *Radius*, and consequently *x p* ☉ as much of that Meridian as appears within the Plane of the Projection. Let the whole *Equator* to be projected ♈ *y* ♎ ♑, and the proper Meridian continued to *a*: Then we have in the Triangle ♈ *a* ☉, *First*, ♈ ☉, the Distance of the *Sun* from the First Point of *Aries* 42° 15′ 13″; *Secondly*, Fig. 32.

a ♈ ☉

æ ♈ ☉ the Inclination of the Planes of the *Equator* and *Ecliptic* to each other, 23° 29′ 00″; and *Thirdly*, ☉ *æ* ♈ = 90 Degrees, a great Circle, as *d p æ*, passing thro' the Pole *p* of the *Equator* cutting the *Equator* at Right-Angles by the Nature of the *Sphere*: And we want ♈ *æ* the *Sun*'s Right Ascension from the First Point of *Aries*; *æ* ☉ his Declination from the *Equator*, the Complement of which to 90° is ☉ *p* his Distance from the Pole; and ♈ ☉ *æ*, the Angle which the proper Meridian makes with the *Ecliptic* in this Place.

And *First*, for ♈ *æ*, or the *Sun*'s Right Ascension at this time: a Line drawn from *p* the projected Pole of the *Equator* to *æ*, will cut off ♈ *a* in the *Ecliptic* Equal to that Arch in the *Equator* which ♈ *æ* represents; by *Coroll.* 1. *Lemma* II. *Sect.* III. *Chap.* II. And if you open your *Sector* to the *Radius* of your Projection, you will find upon your Line of *Chords* that ♈ *a* is nearly 39° 48′ 00″; which is Equal to the *Sun*'s Right Ascension in the *Equator* at that time: And the larger the *Radius* of your Projection, and the more and nicer the Subdivisions on your *Sector*, the more accurately may you determine the *Sun*'s Right Ascension at that time.

Secondly, For *æ* ☉, the *Sun*'s Declination from the *Equator*: 'Tis plain that *p æ* is a projected Quadrant of the proper Meridian *d p æ*, it representing so much of it as is intercepted 'twixt the Pole of the World and the *Equator*; that is, 90 Degrees; and ☉ *u* is a Quadrant of the same, because from the *Sun* to the *Horizon* of the *Disc* is 90 Degrees, as the *Sun* Illuminates one half of the Globe at the same time; therefore *p* ☉ added on both Sides, making ☉ *æ* and *p u* Quadrants of a Great Circle ☉ *æ* is Equal to *p u*: And the projected Great Circle *d p* ☉ being found to be Inclin'd to the Plane of the Projection about 72°, 25′ 00″, by *Coroll.*

Coroll. ad Prop. III. *Sect.* II. *Chap.* II. its Pole which falls within the Plane of the Projection is distant from that of the *Primitive* e, $36°\ 12'\ 30''$; by *Lemma* I. *Sect.* III. *Chap.* II. The Tangent of which to the *Radius* of the Projection laid off from e to $\varkappa$ gives $\varkappa$ the Pole required; from which a Line drawn thro' p to μ gives $f\ \mu$ Equal to that Arch in the *Proper Meridian* which $\varkappa\, p = \odot\, a$ represents, by the *Corollary* refer'd to in the preceding Case; which will be found to be something more than $15°\ 32'\ 00''$.

Thirdly, For $\Upsilon\ \odot\ a$, the Angle which the proper Meridian makes with the *Ecliptic*, or its Alternate which is Equal to it $f \odot \varkappa$: 'Tis plain that $\odot f$ and $\odot\, \varkappa$ being Quadrants, $f\, \varkappa$ is the Measure of the Angle of $f \odot \varkappa$, and $\varkappa\, e$ its Complement to 90 Degrees: In order to find out the Value of which, a Line drawn from $\odot$ thro' $\varkappa$ to λ gives us $\lambda\, d$, which is the Measure of twice the Angle which $\varkappa\, e$ is the Tangent of to the *Radius* of the Projection; by *Prop.* I. *Sect.* III. *Chap.* II. and $\varkappa\, e$ being the Tangent of half the Arch which it represents, that Arch and $\lambda\, d$ are Equal: And $\lambda\, d$ is found upon the *Sector* to be something more than $17°\ 49'\ 00''$; therefore $\varkappa\, e$ represents the same; and $f\, \varkappa = f \odot \varkappa = a \odot \Upsilon$ is something less than $72°\ 11'\ 00''$.

Q. E. F.

Trigonometrical SOLUTION.

'TIS plain that the former way of solving this Problem (and the Reason is the same for all others) cannot come to that Accuracy which an *Astronomer* requires, unless the *Radius* of his Projection is large, and his *Sector* so too, with Nice and Accurate Subdivisions: Tho' it be therefore very pleasant, and with these Advantages

capable of great Exactness, and absolutely necessary when we are not furnished with either *Natural* or *Logarithmical* Tables, yet the *Trigonometrical* Solution by *Spherical* Triangles is the most to be depended upon, as giving us the precise Truth, and not leaving us for any *Minutes* or *Seconds* to Conjectures. If we take therefore the Right-angled *Spherical* Triangle ♈ æ ☉, *Fig.* 32, and place it in *Fig* 33, we have the Angles ☉ ♈ æ, ♈ æ ☉, and the Side ; ♈☉ ; and we want ♈æ, æ ☉ and ♈ ☉ æ: For which in their Order he who is acquainted with Right-angled *Spherical Trigonometry*, will use the following Proportions.

R : *c.S.* ☉ ♈ æ : : *T.* ♈ ☉ : *T.* ♈ æ.
R : *S.* ☉ ♈ æ : : *S.* ♈ ☉ : *S.* ☉ æ.
R : *c.S.* ☉ ♈ : : *T.* ☉ ♈ æ : *c.T.* ♈ ☉ æ.

R : *c.S.* ☉ ♈ æ : :	23°.	29′.	00″.	9. 962452	
T. ♈ ☉ :	42.	15.	13.	9. 958300	
T. ♈ æ.	39.	48.	05.	9. 920752	= ☉is [A. R.
R : *S.* ☉ ♈ æ : :	23°.	29′.	00″.	9. 600409	
S. ♈ ☉ :	42.	15.	13.	9. 827635	
S. ☉ æ.	15.	32.	31.	9. 428044	= ☉is [Dec. Bor.
R : *c.S.* ♈ ☉ : :	42°.	15′.	13″.	9. 869335	
T. ☉ ♈ æ :	23.	29.	00.	9. 637956	
c.T. ♈ ☉ æ.	72.	10.	24.	9. 507291	= L Me- [rid. Cum Eclipt.

Whence we discover that when the *Sun* is in ♉	12°	15′	13″
His Right Ascension is	39	48	05
Northern Declination	15	32	31
Distance from the North Pole	74	27	29
And the Angle which the *Proper Meridian* makes with the *Ecliptic*	72	10	24

Q. E. I.

Scholium 1. By the Converſe of this Problem having the *Sun*'s Right Aſcenſion, and Diſtance from the Pole, we may find his Place in the *Ecliptic:* For then in the Triangle ♈ ☉ *a* (*Fig.* 32.) we have ☉ *a* the Complement of his Diſtance from the Pole, *a* ♈ his Right Aſcenſion, and *a* ♈ ☉ the Inclination of the Two Planes of the *Equator* and *Ecliptic*: And conſequently may eaſily find ♈ ☉, either upon the Projection, or by *Trigonometrical* Calculus.

Schol. 2. After the ſame manner as we proceed in this *Trigonometrical* Solution, the *Aſtronomers* calculate the *Sun*'s Right Aſcenſion, and Diſtance from the *North* Pole of the World to every *Degree* and Minute of the Quadrant ♈ ☉ ♋; and having 'em for that they may eaſily be had for the whole Circumference of the *Ecliptic*. For with reſpect to the *Sun*'s Declination (which ſubtracted from a Quadrant when *North*, and added to it when *South*, gives his Diſtance from the *North* Pole of the World) his greateſt *Northern* Declination is when he is in the firſt Point of *Cancer*, being 23° 29′ 00″; and then his Diſtance from the *North* Pole is 66° 31′ 00″; and that Declination decreaſes from ♋ to ♎ in the *Ratio* in which it decreaſes from ♋ to ♈: ſo that in the *Second* Degree of ♋ it is in the 29th of ♊; and ſo on: And from the *Firſt* Point of *Libra* to that of ♑ his *Southern* Declination increaſes in the ſame *Ratio* as the *Northern* does from ♈ to ♋, (ſo that in ♏ 12° 15′ 13″ his *Southern* Declination is 15° 32′ 31″, and conſequently his Diſtance from the *North Pole* of the World 105° 32′ 31″) till in the *Firſt* Point of *Capricorn* it becomes 23° 29′ 00″, and then his Diſtance from the *North* Pole is 113° 29′ 00″; the Declinations therefore of the *Firſt* Quadrant, with the Title of *Southern*, will ſerve for the *Third* Quadrant of the *Ecliptic*, and theſe decreaſing from

the *Tropic* of *Capricorn* as the *Northern* ones do from the *Tropic* of *Cancer*, the Declinations of the *Second* Quadrant of the *Ecliptic* will serve for the *Fourth*. And in like manner having calculated the *Sun's* Right Ascension to the *First* Quadrant of the *Ecliptic*, you may make it out by bare *Addition* and *Subtraction* to other *three*: For if the *Sun* had appear'd in the *Fourth* Quadrant of the *Ecliptic* as remote from the *First* Point of *Aries* as he is in ♉ 12° 15′ 13″; *viz.* in ♒ 17° 44′ 47″; the Right Ascension of this Point in ♉ will be found upon Calculation 45° 16′ 25″, which taken from 360, the Right Ascension of the Last Point in ♓, leaves 314° 43′ 35″ for the Right Ascension of ♒ 12° 15′ 13″. If the *Sun* were in the *Second* Quadrant as remote from the *First* Point of *Libra* as he is in the Projection from the *First Point* of *Aries*, *viz.* in ♌ 17° 44′ 47″; the Right Ascension of this Point in ♉, *viz.* 45° 16′ 25″ taken from 180 (the Right Ascension of the *First* Point of *Libra*) leaves 134° 43′ 35″ for the Right Ascension of ♌ 12° 15′ 13″. And if the *Sun* were in the *Third* Quadrant as remote from the *First* Point of *Libra* as he is in the Projection from the *First* Point of *Aries*, *viz.* in ♏ 12° 15′ 13″; the Right Ascension of that Point in ♉, *viz.* 39° 48′ 05″, added to 180 gives 219° 48′ 05″ for the Right Ascension of ♏ 12° 15′ 13″. And after the same manner may we have the Right Ascension of every Point of the *Ecliptic*.

Schol. 3. As *x p* is prov'd Equal to ⊙ *æ* in the former of the Two Solutions of this *Problem*; so it holds Universally that the Illuminate Part of the *Proper Meridian* intercepted 'twixt the Pole of the World and the *Horizon* of the *Disc* is Equal to the *Sun's Northern* Declination in the *Summer-Months*, for the same reason: And the Obscure Part of the same *Meridian* 'twixt the Pole of the World

World and the *Horizon* of the *Disc* is Equal to his *Southern* Declination in the *Winter Season*. This Part of the *Proper Meridian* is call'd the *Reflection*, and is of Use in the Solution of several of the more *Particular* Problems.

PROBLEM II.

Having the Longitude and Latitude of a Fix'd Star, *To find the Right Ascension, and Declination.*

SOLUTION *upon the Projection.*

LET the *First Star* in *Aries* be that whose Right Ascension, and Declination we want: Its Longitude from the Vernal *Equinoctial* Point (in the beginning of the Year 1711) is ♈ 29° 00′ 50.″; according to Monsieur *de la Hire*'s Tables; and its Latitude 7° 09′ 17″, *North*. The Chord therefore of 29° 00′ 50″, to the *Radius* of the Projection, set off from ♈ to ϰ gives ♈ ϰ its Longitude; and ♏ ε ϰ is the Representative of its Circle of Longitude; by *Prop.* I. *Sect.* II. *Chap.* II. And the Tangent of half the Complement of its Latitude set off from ε to ✱, gives ✱ the Place of the *First Star* in ♈, by *Prop.* I. *Sect.* III. *Cap.* II. And as the Circles of Right Ascension pass thro' both the projected Poles of the World, and the Stars whose Right Ascension they determine: If we find out the *South* Pole by *Lemma* I. *Sect.* III. *Cap.* II. and thro' that and the Points *p* and ✱ strike a Circle by 5. IV. *Euclid*; we shall find the Circle λ *p* ✱ θ to be the proper Circle of Right Ascension to the *First Star* of *Aries*; having χ for its Centre; and ε χ being the Tangent of its Inclination to the Plane of the Projection, to the *Radius* ε ♑ by *Prop.* III. *Sect.* II. *Cap.* II. which being [illegible] its Pole ω may be found also, by the *Lem-*

ma now refer'd to; and a Line drawn from that Pole thro' ✱ to ♑, will give μ ♉ in the *Ecliptic* Equal to that Part of the *Star*'s Circle of Right Ascension which γ ✱ represents; and ω ♌ θ will cut off μ ♌ Equal to what is represented by γ ✱ θ; from which γ ✱ being taken, there remains ✱ θ the *Northern* Declination of the *First Star* of ♈ at this time: And a Line drawn from *p* thro' Φ to θ gives us ♈ Ψ Equal to that Part of the projected *Equator* which ♈ θ represents: That is, the Right Ascension of this Star. Both by *Coroll.* I. *Lemma* II. *Sect.* III. *Cap.* II. The former being found, even upon this small Projection, 17 *Degrees* and about 49 *Minutes*; and the Latter 24 *Degrees*, and above 17 *Minutes*.

Q. E. F.

Trigonometrical SOLUTION.

The *Star* being projected as before, 'tis plain that *e* ϰ is its Circle of Longitude in the *Ecliptic*; and its Longitude being 29° 00′ 50″, ♈ ϰ must be so much: And in the Triangle ♈ ϰ δ, having the Side ♈ ϰ 29° 00′ 50″; the Angle ϰ ♈ δ, 23° 29′ 00″, and the Angle at ϰ Right, we may have ♈ δ, ϰ δ, and ♈ δ ϰ, by the following Proportions:

R : c.S. ϰ ♈ δ : : c.T ♈ ϰ : c.T. ♈ δ.
R : S. ♈ ϰ : : T. ϰ ♈ δ : T ϰ. δ.
R : c.S. ♈ ϰ : : S. ϰ ♈ δ : c.S. ♈ δ ϰ.

R : c.S. ϰ ♈ δ =	23° 29′ 00″.	9. 962452
c.T. ♈ ϰ =	29. 00. 50.	10. 255999
c.T. ♈ δ. =	31. 09. 42.	10. 218451

R : S.

R: S. ♈ ✶ = 29°. 00′. 50″. 9. 683761
T. ✶ ♈ δ = 23. 29. 00. 9. 637956
T. ✶ δ. 11. 53. 58. 9. 321717

R: c.S. ♈ ✶ = 29°. 00′. 50″. 9. 941760
S. ✶ ♈ δ = 23. 29. 00. 9. 600409
c.S. ♈ δ ✶. 69. 36. 22. 9. 542169

Hence we have ♈ δ, 31° 09′ 42″; ✶ δ, 11° 53′ 58″; and ♈ δ ✶, 69° 36′ 22″; and ✶ ∗ being 7° 09′ 17″, δ ∗ must be 19° 03′ 15″: Therefore in the Triangle δ ∗ θ, having θ δ ∗ = ♈ δ ✶ 69° 36′ 22″; the Angle at θ Right; and δ ∗ 19° 03′ 15″: We may have δ θ call'd the *Prostaphæresis* of Right Ascension, and θ ∗ the Star's *Northern* Declination by the following Proportions.

R: S. ∗ δ θ :: S. ∗ δ : S. ∗ θ.
R: c.S. ∗ δ θ :: T. ∗ δ : T. δ θ.

R: S. ∗ δ θ = 69°. 36′ 22″. 9. 971887
S. ∗ δ = 19. 03. 15. 9. 513832
S. ∗ θ. = 17. 49. 05. 9. 485719

R: c.S. ∗ δ θ = 69°. 36′. 22″. 9. 542168
T. ∗ δ: = 19. 03. 15. 9. 538304
T. δ θ = 06. 51. 46. 9. 080472

So that for the Angle ♈ p θ, or the Arch ♈ θ, representing the Right Ascension of the *First Star* of *Aries* at this Time; the *Prostaphæresis* θ δ, 06° 51′ 46″ must be subtracted from δ ♈, call'd the Competent Right Ascension, 31° 09′ 42″: Which being done, it will leave the True Right Ascension of this *Star* at this time, 24° 17′ 56″; its *Northern* Declination being 17° 49′ 05″; and

consequently its Distance from the *North* Pole of the World 72° 10′ 55″.

Q. E. I.

Scholium 1. By the Converse of this *Problem*, having the Right Ascension and Declination of a *Fix'd Star*, we may find out its Longitude and Latitude. For admitting the Right Ascension of the *First Star* in *Aries* to be 24° 17′ 56″, in the beginning of the Year 1711; and its *Northern* Declination 17° 49′ 05″; we have in the Triangle ♈ θ ŋ ♈ θ, 24° 17′ 56″, θ ♈ ŋ 23° 29′ 00″, and the Angle at θ Right: Whence we discover ♈ ŋ, the Competent Longitude, 26° 12′ 34″; ŋ θ, 10° 08′ 10″; and ♈ ŋ θ 68° 42′ 16″: And taking ŋ θ from the Declination known 17° 49′ 05″, ŋ ∗ is 7° 40′ 55″. Therefore in the Triangle ∗ ϰ ŋ Right-angl'd at ϰ, having ŋ ∗, ∗ ŋ ϰ = ♈ ŋ θ, and the Angle at ϰ Right; we find ϰ ∗ the Latitude of the *First Star* in *Aries* 7° 09′ 16″ *North*; and ϰ ŋ the *Prostaphæresis* of Longitude, 2° 48′ 17″; which added in this case to ♈ ŋ the Competent Longitude, 26° 12′ 34″, makes the Real Longitude of this *Star* at this time, ♈ 29° 00′ 51″. both within one *Second* of a *Degree* of what we assum'd in the *Problem*.

Schol. 2. Whereas, by reason of the Recess of the *Equinoctial* Points 50 *Seconds* Yearly, the *Fix'd Stars* seem to move forward so much from the *First* Point of *Aries* in the same Space of Time, in their respective Parallels of Latitude; thereby increasing their Longitude 50 *Seconds* Yearly, but keeping their Latitude always the same: And whereas, as they vary their Longitude, their Right Ascension and Declination must vary in some assignable Proportion. By the Method observ'd in the Solution of this *Problem*, it is easy for any one to discover how much any assignable *Fix'd Star* varies its Right Ascension and Declination in any Compass of Time. Let it be requir'd

quir'd therefore to answer this Question with Relation to the *First Star* in *Aries*, for 100 Years to come from the beginning of the Year 1711: We have already assign'd its Place at this Time to be in *Aries* 29° 00′ 50″; therefore in the beginning of 1811 it will be in ♉, 00° 24′ 10″; 1° 23′ 20″ being what it increases in Longitude in 100 Years. Therefore (supposing it to have mov'd in its Parallel of Latitude from ∗ to ·⋇·. its new Circle of Longitude being s ·⋇· ♉, and that of Right Ascension p.⋇. ♄) in the Triangle ♈ ♉ ☿, Right-angled at ♉, we have ♈ ♉, the *Star's* new Longitude from the *Vernal Equinox*, 30° 24′ 10″; ♉ ♈ ☿, 23° 29′ 00″; and the Right Angle at ♉. By which we are inabled to find ♈ ☿ the Competent Right Ascension to this New Longitude, which upon Calculation appears to be 32° 36′ 32″, ♉ ☿ ♈, 69° 53′ 56″; and ♉ ☿, 12° 24′ 02″; to which Last ♉ ∗ being added, 7° 09′ 17″, it makes ☿ ·⋇·, 19° 33′ 19″. Farther, in the Right-angl'd Triangle ♄ ·⋇· ☿, Right-angl'd at ♄; having ☿ ∗, 19° 33′ 19″, ♄ ☿ ·⋇· 69° 53′ 56″; and the Angle at ♄ Right; we find ♄ ·⋇· the *Star's* New Declination, 18° 19′ 12″; and consequently its Variation of Declination in 100 Years 00° 30′ 07″ nearer the *North* Pole of the World: And ♄ ☿, the *Prostaphæresis* of Right Ascension 6° 57′ 36″; which taken from 32° 36′ 32″, the Competent Right Ascension, leaves 25° 38′ 56″ for the New Right Ascension of this *Star*: And consequently 1° 21′ 00″ is its Variation of Right Ascension in 100 Years to come from the beginning of 1711. And after the same manner are the Variations of Right Ascension and Declination of all the other *Fix'd Stars*, resulting from the Recess of the *Equinoctial* Points, calculated to this or any other Number of Years by the *Astronomers*; who in their Catalogues of the *Fix'd Stars*,

 give

give us the Right Ascension, and either Declinatiohor Distance from the Pole, together with the Longitude and Latitude of each *Star* to a determinate *Epocha*; and their Variation of Right Ascension and Declination, or Distance from the Pole, to 10, 20, 50, or 100 Years; in order to inableus to make them out to any intermediate Times. Mr. *Flamstead*, indeed, in his Excellent Catalogues of the *Fix'd Stars*, with which he has inrich'd the World, gives us the Variations of Right Ascension and Declination of all the *Stars* to one *Degree* of their apparent Annual Motion from the Places they are found in at the beginning of the Year *1690*; which they complete in 72 Years: And according to him, the *First Star* of *Aries* is in the beginning of the Year 1711, in *Aries*, 29° 00′ 10″; 40 *Seconds* short of Mr. *de la Hire*'s Place: and his Variation of Right Ascension of this *Star* in 72 Years is 00° 58′ *16*″; and the Variation of its Declination at the same time is 00° 21′ 43″; both which agree accurately well with the preceding Determinations. The Intelligent Reader must observe, that the Distance of the *First Star* of *Aries* from the *North* Pole of the World will be less 100 Years hence than it was in the beginning of 1711. by 00° 30′ 07″; its *Northern* Declination increasing so much in that time; and it will continue to increase for a great many 100 Years more, till that *Star* is found in the *First* Point of *Cancer* where its Latitude ♋ ♀, 07° 09′ 17″ deducted from ♋ *p* which is *66*° 31′ 00″, leaves its Distance from the Pole only 59° 21′ 43″; its *Northern* Declination being then 30° 38′ 17″: And from this Point to that of ♑, which it will be a great many Thousands of Years in running, its Distance from the *North* Pole must visibly increase; till upon that Point it becomes 106° 19′ 43″; from thence decreasing again till it

it returns to the *First* Point of *Cancer*. The same may be said of all the other *Fix'd Stars* in this *Northern* Hemisphere, excepting those within and near the *Arctic* Circle, which the Reader will find upon Calculation to be subject to more Alterations; and of those of the *Southern*, with respect to the *North* Pole; and the contrary of both the *Northern* and *Southern*, with respect to the *South* Pole of the World. As every one may see who will project the *South* Pole, and observe the Variations of the Declinations of the *Fix'd Stars*, with respect to it.

Schol. 3. The Variation of Right Ascension and Distance from the Pole of the *First Star* in ♈ might otherwise have been had from the Triangle *ep*·♓.; in which are given, *ep* the Distance of the Two Poles of the *Equator* and the *Ecliptic*, *e*.♓. the Complement of the *Star's* Latitude, and *pe*·♓. the Difference 'twixt its New acquir'd Longitude and that of the *First* Point of *Cancer*: From whence we may have *p*.♓. the *Star*'s New Distance from the Pole, and *ep*·♓. the *Star*'s New Right Ascension from the *First* Point of *Capricorn*: And consequently the Variation of its Right Ascension and Declination in 100 Years. After the same manner with the same *Data* we may get those of all the other *Fix'd Stars*. And if we calculate them this and the former Way, and the Determinations agree we may safely conclude that they are just. By this Triangle the Right Ascension of the *First Star* in *Aries*, in the beginning of 1711, is 25° 38′ 55″; and its *Northern* Declination 18° 19′ 12″. Which confirm the Determinations from the other Triangles.

Schol. 4. The seeming Motion of the *Fix'd Stars in Consequentia*, or according to the *Series* of the *Signs*, which arises from the Recess of the *Equinoctial* Points 50 *Seconds* Yearly, is not a New

Discovery in *Astronomy* (tho' it be now better fix'd than it was formerly as to its *Direction* and Annual *Quantity*) but the first Suspicion of it is at least as Old as *Hipparchus's* time: For *Ptolemy* tells us (in his *Mag. Const. Lib.* 7. *Chap.* I.) that *Hipparchus*, upon comparing his *Observations* about the Places of the *Fix'd Stars* with those of *Aristyllus* and *Timocharis*, suspected that at least those near the *Ecliptic* did move from time to time *in Consequentia*: And *Ptolemy* himself went farther; for, upon comparing his own *Observations* with those of *Hipparchus* and others, he found himself necessitated to assert that all the *Fix'd Stars* had this Motion; and that in particular the *Direction* of this Motion was in Circles Parallel to the *Ecliptic*; the *Quantity* being (as far as his Materials could inable him to judge) about *One Degree* in 100 Years. *Albategnius* after him, from his own *Observations* upon *Cor Leonis*, compar'd with those made by *Menelaus* 785 Years before upon the same *Star*, makes the *Fix'd Stars* to move according to the *Series* of the *Signs One Degree* in 66 Years. *Ulugh Beigh* afterwards allows them 70 Years to *One Degree*. And later *Astronomers*, from comparing the most Ancient with Intermediate and Modern *Observations*, determine this apparent Motion of the *Fix'd Stars* to be Equable and Uniform, in Circles Parallel to the *Ecliptic*, and for Annual Quantity nearly the same as is assign'd them by *Ulugh Beigh*: *Tycho Brahe* allowing them 51 *Seconds* Yearly; and, to mention no more, the greatest Master of *Astronomy* of the present Age, Mr. *Flamstead*, precisely 50 *Seconds*. The Physical Reason of the Annual Recess of the *Equinoctial* Points, from whence this seeming Motion of the *Fix'd Stars* proceeds, I have not room to produce in this Place. Every one knows it who is sufficiently acquainted with Sir *Isaac Newton's* *Principia*;

Prinoipia; to which those who are ignorant of it must be refer'd: Or, to Dr. *Gregory's Astronomy*, *Lib.* I. *Prop.* 64. I shall only observe farther here, that for some time the most Ancient *Astronomers* did not believe this Apparent Annual Motion of the *Fix'd Stars* Equable and Uniform, and regularly continued on from time to time; but that rather after such a Progress they return'd back again by the Way they went, to their old Places; and for the *Solution* of this Motion, they invented what they call'd the *Ninth Sphere*; and that when they found their Error in this Point they added a *Tenth*, to solve their Equable Continued Motion *in Consequentia*, &c. But upon this Subject the Reader may consult *Kepler's Epit. Astronom. Copernicum*, Lib. VII. Pag. 906, 907, &c. *Edit.* Lentiis *ad Danubium*.

Schol. 5. In the *First* Triangle ♈ ɥ θ made Use of in the *First Scholium* to this *Problem*, we have the Angle ♈ ɥ θ, or its Alternate ✱ ɥ ϰ, which the *Meridian* passing thro' the *First Star* in *Aries* makes with the *Ecliptic*: And if that *Star* had been the *Moon*, or any other of the *Planets*, with the same *Data*, it might have been got after the same manner; and that not only for that Point, but for every Point of her *Orbit*, if her *Theory* is known. And as the Angle ♈ ☉ *a* is got in the preceding *Problem*, after the same manner may it be had to every Point in the *Ecliptic*, and annex'd to the *Tables* of the *Sun's* Right Ascensions and Declinations; as it is frequently done.

PROBLEM

Problem III.

Having the Right Ascension and Declination of Two Fix'd Stars, *and the Distance of a* Third, *whether* Fix'd Star *or* Planet, *from each of them; to find out the Right Ascension and Declination of that* Fix'd Star *or* Planet.

Solution.

THERE are *Three* Cases in this *Problem*: *First*, when the *Star* or *Planet* has *more* Right Ascension than either of those *Stars* from whence its Distance is taken: *Secondly*, when it falls betwixt them: And, *Thirdly*, when it has *Less* Right Ascension than either of them.

Case 1. If the *Moon* (for Instance) has more Right Ascension than either of the *Fix'd Stars* from whence her Distance is taken, then, in the Scheme
Fig. 34. annex'd, (1.) Let ☽ represent the *Moon*, whose observ'd Distance is given at a determinate Time from *a* and *b* Two *Fix'd Stars*, whose Distances from the *North* Pole of the World *a p* and *b p* are given; *a q* being suppos'd the *Equator*, and *p* its *North* Pole; and their Right Ascensions being given, the Difference of their Right Ascensions *b p a* will be given also. Therefore in the Oblique-angl'd Triangle *p b a*; having *p b* the Complement of the Declination of the *Star* at *b*, *p a* the Distance of the *Star* at *a* from the *North* Pole of the World, and *b p a* the Difference of the Right Ascensions of the Two *Stars*; we may find out *b a* the Distance of the Two *Stars*, and the Angle *p b a*: Farther, in the Triangle *b a* ☽, having the Sides *b* ☽ and *a* ☽, the observ'd Distances of the *Moon* from the *Stars* at *a* and *b*; and having got *a b* the Distances of those *Stars* from each other, we may find

find out *a b* ☽; which *subtracted* from *a bp* leaves *p b* ☽ known: And *Lastly*, in the Triangle *b p* ☽, having *p b*, *b* ☽ and *p b* ☽, we may find out *p* ☽, the *Moon's* Distance from the *North* Pole of the World, which if less than 90 *Degrees* the Complement to it is her Declination *North*, if greater the Excess is her Declination *South*; and *b p* ☽, the Additional Difference of Right Ascension 'twixt the *Star* at *b*, and the *Moon* at the Time of Observation.

Case 2. If the *Moon* is 'twixt the Two *Stars*, (2.) Having got the Angle *p b a* as before, with the Side *a b*; and in the Triangle *a b* ☽ having found out the Angle *a b* ☾, *adding* that to *p b a* we have *p b* ☽; and consequently may have *p* ☽, and *b p* ☽; which Last added (as before) to the Right Ascension of *b*, gives the *Moon*'s Right Ascension.

Case 3. If the *Moon* has less Right Ascension than either of the *Stars*, (3.) Having got the Angle *p a b*, from *p a*, *p b*, and *a p b* given; and in the Triangle *a b* ☽, having found out *a* ☽ and the Angle *b a* ☽, that taken from *p a b* gives us *p a* ☽: And having *p a* ☽, *a p* and *a* ☽, we may have *p* ☽ the Distance of the *Moon* from the Pole; and *a p* ☽, the Difference of the Right Ascensions of the *Star* at *a* and the *Moon*; which, in this Case, *subtracted* from the Right Ascension of the *Star* at *a* leaves the *Moon*'s Right Ascension.

Q. E. I.

Scholium 1. If the Longitude and Latitude of the Two *Fix'd Stars* had been given, instead of their Right Ascension and Declination; the *Moon's* Longitude and Latitude might have been found by the same Method in which we have directed how to find her Right Ascension and Declination: As every one may see in the Schemes refer'd to, if we

we will call *p* the *North* Pole of the *Ecliptic*, and *æ q*, the *Ecliptic* itself. And whereas, by reason of the great Density of the *Atmosphere* which surrounds our *Earth* in Comparison of the Infinitely Fine *Æther*, which after some few Miles fills up those Immense Spaces 'twixt us and the *Fix'd Stars*, the Rays of Light from each *Star* are so *refracted* as to represent it at all Times something nearer our *Vertex* than it actually is; when it is not upon the *Vertex* itself; and that the more, the nearer the Object to the *Horizon*; so that at the *Horizon* itself a *Star* is *refracted* (according to Mr. *Flamstead's* Table of *Refractions*) 33′ 00″; at 80 *Degrees* from the *Vertex* 04′ 33″; at 70 *Degrees* 2′ 14″; at 60 *Degrees* 1′ 23″; at 50 *Degrees* 0′ 58″; at 40 *Degrees* 0′ 40″; and from thence about as many *Seconds* as it is *Degrees* from the *Vertex*; hence it comes to pass that the *observ'd* Distances of any Two *Stars* from each other, or a Third *Star* or *Planet* are *contracted* more or less, in Proportion as they are nearer to or more remote from the *Horizon*. Thus (in the *Second* Scheme at *Fig*. 34.) by reason of the *Refractions* the *Moon* does not really appear at her due Distance from the *Vertex* in the Point *Luna*, but elevated more or less, in Proportion to her Distance from the *Horizon*, towards *v* the *Vertex*; so as to be seen, for Instance, in the Point *c*; and the *Star* at *b* appears to be at *d*; making thereby *d c* the Apparent, or *Observ'd* Distance 'twixt the *Moon* and this *Star* less than ☽ *b* their Real Distance; which, if not allow'd for, must corrupt the Right Ascension and Declination, Longitude and Latitude of the *Moon* at the time of *Observation*; and that the more, the nearer the *Moon*, or the *Stars* you observe her from, or any, or all of 'em to the *Horizon*. In order to remedy this, the *Astronomers* have deduc'd from *Observations*, or otherwise, *Tables of Refractions* to every

Degree

Degree of the *Quadrant* from the *Vertex*; so that having the *Moon's* Apparent Height at *c* you have the *Refraction*, which taken from the Visible Altitude leaves her at the Point ☽; and having the Apparent Height of the *Star* at *d*, the Correspondent *Refraction* taken from it leaves it at the Point *b*: And having these, that is, in the Triangle *v b* ☽ having *v b* and *v* ☽, and the Angle *b v* ☽ their Difference of *Azimuths* at the time of *Observation*, which will be taught how to be had under the Head of the *Particular Problems*, you may have *b* ☽ their Real Distance, the Difference of which from *c d* their Apparent or *Observ'd* Distance is what is call'd the *Contraction* by *Refractions*, which when taken from Tables already made is *added* to the Apparent Distance to give the Real. For this also the *Astronomers* make Tables to several Distances from the *Vertex*, and to as many Distances of the *Stars* from each other as they have usually occasion for; in order to be more expedite in their Deductions of the Places of either *Fix'd Stars*, the *Moon*, or any other of the *Planets*, from these Kinds of Observations.

Schol. 2. If we *observe* the *Moon's* Distance from only One *Star* whose Right Ascension and Declination are known, and take their Difference of Right Ascensions from their Observ'd Times of passing over the *Meridian*, we may easily discover the *Moon's* Declination: For (in the *First* Scheme at *Fig.* 34.) having the *Moon's Observ'd* Distance from the *Star* at *a*, *a* ☽, and the Difference of their Right Ascensions *a p* ☽, and the *Star's* Distance from the Pole *a p*; in the Triangle *a p* ☽, we may easily discover *p* ☽ the Complement of the *Moon's* Declination, and consequently the Declination itself. Or having the *Moon's* Declination with the other Requisites, at this time we may easily find out her Right Ascension: And so for any other

of

of the *Planets*. And whereas by reason of the *Moon's* swift Motion according to the *Series* of the *Signs*, she must alter her Right Ascension considerably in a short time, the *Star* which we take her Distance from ought to be chosen as near her as we can, that the Variation of her Right Ascension may be as little as possible from the time that the *Star* passes to her *transiting* the *Meridian*, or from her passing to that of the *Star*; which must be allowed for, accordingly as she or the *Star* passes *First*, from her *Mean Motions* and *Theory* known. The other *Planets* move prodigiously slower, so that we are not so much confin'd as to the Choice of single *Stars* wherewith to compare them: And as to either the *Moon* or any other of the *Planets*, when we would discover their Places by *observing* their Distances from single *Stars*; the best way is to take their Declination upon a *Quadrant* fix'd in the Plane of the *Meridian* of the Place we are in, when they come upon that Plane; and at the same time their Distance from the *Star* we pitch upon by a *Sextant*, correcting that Observ'd Distance according to the Method deducible from the preceding *Scholium*, from whence we may have their Right Ascension at that Time; and from that and their *Observ'd* Declination their Longitude and Latitude: Always taking care, according to their Altitude, to make proper Allowances for *Refractions*.

Schol. 3. This and the two foregoing *Problems*, with the *Solar Theory* given, and the Latitude of the Place we are in, are the great Foundations the *Astronomers* build upon in deducing the Right Ascensions and Declinations, Longitudes and Latitudes of all the *Fix'd Stars* in the Heavens, and the *Theories* of the *Planets*, their Motions, Orbits, Periodical Times, Conjunctions, Oppositions, &c. For having the *Sun's* Theory we have its Place in the

the *Ecliptic* to any assignable Time, and having that we may have his Right Ascension to the same Time; and having the *Sun*'s Right Ascension to to any Day at Noon, that is when he is observ'd to come upon our *Meridian*, by a *Quadrant* or *Semi-Circle* fix'd in its Plane, if we have an exact *Equatorial* Pendulum-Movement, or a good common Clock, with a Table for the Reduction of *Solar* to *Equatorial* Time, set at that Time to the Hour of *Twelve*; and observe the next Night when any Notable *Fix'd Star* comes to the *Meridian*; the time elaps'd 'twixt its Culmination and that of the *Sun* turn'd into *Degrees* and *Minutes* of the *Equator* gives us the Difference of their Right Ascensions, which added to the *Sun*'s Right Ascension gives us that of the Observ'd *Star*: And after the same manner may we have that of all the *Stars* in the Firmament which appear above our *Horizon*: And having the Latitude of the Place we are in, we have its Complement to 90 *Degrees*, the Elevation of the *Equator* above the *Horizon*: And having that, and the *Meridional* Altitude of any *Star* corrected by Allowance for *Refractions*, we may have its Declination, and consequently its Distance from the *North* Pole of the World; and from them its Longitude and Latitude: And after the same manner may we get so many Places of the *Planets* as will enable us to determine their *Orbits*. Or *observing* the Right Ascensions and Declinations of some of the Principal *Fix'd Stars* after this manner, we may have those of the rest, and all the *Planets* in any Points of their Orbits, by *observing* with a *Sextant* their Distances from such *Fix'd Stars* whose Right Ascensions and Declinations are known; and thence by the Method observ'd in the Solution of the present *Problem* deduce their Right Ascensions and Declinations, Longitudes and Latitudes. The

I *Inclination*

Inclination of the Planes of the *Equator* and *Ecliptic* may be had by *observing* the *Meridional* Heights of the *Sun* for several Days before and after the *Summer* and *Winter* Solstices, by which finding out his Greatest and Least *Meridional* Heights, the latter taken from the former will leave an Arch which bissected will give us the *Inclination* of those Planes; and that *half* added to the Least *Meridional* Height of the *Sun* will give us the Elevation of the *Equator* above the *Horizon*, the Complement of which to 90 *Degrees* is the Latitude of the Place where our *Observations* are made.

Schol. 4. In order to be able to get the Right Ascensions of the *Fixed Stars* from their Transits over the *Meridian*, we must *First* find out the *Meridian-Line* to our Place of *Observation*, in order to *Fix* our Instrument by which we observe their Transits in the Plane of it. And this *Meridian-Line* is of so great Use in *Astronomical Observations*, that the *Astronomers* have spar'd no Pains to investigate the most accurate Ways by which we may determine it. Of all those which they have thought of, I know none easier or more to be depended upon than this following. Having a Table, or any other Plane you are minded to draw your *Meridian-Line* upon, fix'd in the *Horizontal* Plane, or one Parallel to it; and a *Quadrant* of Three or Four Foot *Radius* well graduated, and provided with an *Optical Tube*, and *Telescopic* Sights; so fastened in its Centre to the Edge of that Plane, as that it may turn upon that Centre to whatever Part of the Plane in which you shall have occasion to make use of it: Take the Height exactly of any Notable *Fix'd Star* which has considerable *Northern* Declination, Two or Three Hours before it comes to the *Meridian*; and along the Edge of your *Quadrant* which touches the Plane

Plane draw a Line to what Length you pleaſe from the Centre of the Inſtrument, ſo as that it may coincide with the Plane of the *Azimuth* Circle which the *Star* is upon at this Time of *Obſervation*: After this *Star* has paſs'd the *Meridian*, watch when it comes to the ſame Height preciſely which you *obſerv'd* it to have before it came thither; and when you have found this out, draw a Line as before from the Fix'd Centre of your *Quadrant* along the Edge of it which touches your *Horizontal* Plane, ſo as that Line coincide with the Plane of the *Azimuth-Circle* which the *Star* is now upon: Upon the Point of Interſection of theſe Two Lines, which is the ſame with that where you fix the Centre of your *Quadrant*, place one Point of your Compaſſes; and, opening them to what Diſtance you pleaſe, ſtrike a Circle biſſecting that Arch of it which is intercepted 'twixt the Two Lines: A Line drawn from the Point of Biſſection to the Centre of the Circle will be the *Meridian-Line* requir'd. If we would uſe the *Sun* inſtead of a *Fix'd Star*, we muſt *obſerve* him at a time when he alters Little or Nothing his Declination in Four or Five Hours Time, otherwiſe our Determination will be faulty; that is, at or near the *Solſtices*; and the *Summer Solſtice* is the beſt, becauſe then he alters his Altitude the moſt in paſſing from one to another *Azimuth*; and conſequently it is the eaſieſt to find when he is preciſely at the ſame Height, before and after he paſſes the *Meridian*: Which is the reaſon alſo why I chuſe a *Fix'd Star* with *Northern* Declination, in our Latitude, for this purpoſe. And if we make uſe of the *Sun*, we may accompliſh what we deſire by erecting a *Style* perpendicular to our *Horizontal* Plane upon the Center of ſeveral Concentrical Circles; and finding when and where the Shadow of the *Vertex* of this *Style* touches the

Circumference of any one of these Concentrical Circles, and drawing Lines from the Centre to the Points of Contact, and bissecting the Arch intercepted 'twixt those Points; a Line drawn from the Point of Bissection to the Centre will be the *Meridian-Line* requir'd. This Process needs no Explication to any one who knows what we mean by the *Meridian-Line*: For when the *Sun* or *Star* made use of are in the highest Point of their Diurnal Parallels, they are upon the *Meridian* itself: And when they are on Points equally distant from the *Horizon* on each Side of the *Meridian*, the *Meridian* is in the Middle 'twixt the *Azimuths* which they then are upon: Which may consequently be easily found in the Way now deliver'd.

SECT.

SECT. II.

Sect. II.

The Solution of such Problems resulting from the Diurnal Phænomena, *as require our Knowledge of the* Latitude *of the Place we are in, in order to determine them.*

PROBLEM I.

Having the Sun's *Place in the* Ecliptic, *and the* Reflection, *or Declination of the* Sun *at that Time together with the Latitude of the Place we are in; to find out the Time of his Rising and Setting, and the Length of the Day and Night; together with the* Amplitude *of the* Rising Sun *from the* East *and the* Setting *from the* West; *and that of the* Path *of our* Vertex *in the* Horizon *of the* Disc.

SOLUTION *upon the Projection.*

LET these Particulars be requir'd to the Latitude of *London*, 51° 32′ 00″ *North*. The *Path* of its *Vertex* in this Projection is *m o n y*, *m* being the Place where the *Sun* rises, or the *Vertex* in its Diurnal Motion passes out of the *Obscure* into the *Illuminate* Part of the *Disc*, *o* the Place where it is at *Noon*, and *n* the Place where the *Sun* sets, or the *Vertex* passes out of the *Illuminate* into the *Obscure* Part of the *Disc*; so that *m o n* is the Diurnal, and *n y m* the Nocturnal Part of the Path of the *Vertex* of *London*. *First*, therefore, for the time of the *Sun*'s Rising and Setting, *&c.* If we can find the Value of *n y m*, we have the Length of the Night at *London* to this Place of the *Sun*, and consequently that of the Fig. 32.

the Day, and the Times of *Sun*-Rising and Setting: And the Value of that Arch may be found by projecting a Circle Parallel to the *Ecliptic*, equally distant from its *South* Pole as the Parallel of *London* is from the *North* Pole of the Globe; according to the Directions of *Prop.* IV. *Sect.* II. *Cap.* II. And if we lay a Ruler upon *p* and *n* reaching to the New projected Circle, and mark the Point where it intersects that Circle, and likewise upon *p* and *m* reaching to the same Circle, and mark the New Point of Intersection; the Arch intercepted 'twixt these two Points measured upon the *Line of Chords* of the *Sector*, open to the *Radius* of the New projected Circle, gives us what *n y m* represents: By *Schol.* and *Coroll.* 1. *ad Lemma* II. *Sect.* III. *Cap.* II. which turn'd into Time gives us the Lengh of the Night, the Complement of that to 24 Hours the Length of the Day, the Length of the Night bissected the Time of the *Sun*-Rising, and deducting half the Length of the Night from 12 Hours we have the Time of *Sun*-Setting. The *Amplitude* of the *Path* of the *Vertex* of *London* in the *Horizon* of the *Disc*, which is here represented by *x n* or *x m*, may be found out by discovering the Value of the Arches of a great Circle of the *Sphere* which these represent according to the Directions of *Coroll. ad Prop.* I. *Sect.* III. *Cap.* II. And for the *Amplitude* of the Rising-*Sun* from the *Eastern Azimuth* (and the opposite Point in the *Horizon* is his Setting *Western Amplitude*) in the Triangle *p m* ☉ we have the Side *m* ☉ the Distance of the *Sun* from the *Vertex* at his Rising, which is a Quadrant of a *Vertical* Circle; and striking another great Circle thro' the Points *p* and *m*, and the Projected *South* Pole of the World, the Angle intercepted 'twixt them, *viz.* *p m* ☉, is the *Azimuth* of the Rising-*Sun* from the *North*: In order to find out the Value of which, the Circle

cle *m p* being compleated, and a Quadrant of it being taken off from *m* on the Side of *p*, which may be done (after having found its *Inclination* to the Plane of the Projection, by *Coroll. Prop.* III. *Sect.* II. *Cap.* II. and its Pole which falls within that Plane, by *Lemma* I. *Sect.* III.) by *Coroll.* 1. *Lemma* II. *Sect.* III. *Cap.* II. And a Circle being struck thro' the *Quadrantal* Point found, the Point ⊙, and (as all Great Circles of the *Sphere* must cut the Plane of the Projection in Two Opposite Points) its Opposite Point *d*; and its *Inclination* to the Plane of the Projection and Pole within that Plane being found; its Arch intercepted 'twixt the *Quadrantal* Point found, and the Point ⊙, may easily be measur'd by following the Directions of *Coroll.* 1. *Lemma* II. *Sect.* III. *Cap.* II. And having that Arch, or the Value of the Angle *p m* ⊙, we have the *Sun's Azimuth* from the *North*, the Complement of which to 90 *Degrees* is his *Amplitude* from the *East*. *Q. E. F.*

Trigonometrical SOLUTION.

IN the Triangle *m p x* Right-angl'd at *x* we have *m p* the Distance of the Pole from the *Vertex* 38° 28′ 00″; *p x* = ⊙ *æ*, 15° 32′ 31″; and we want *m p x* the Angle which the *Proper Meridian* makes with that which passes thro' the *Vertex* at *Sun*-Rising, which turn'd into Time gives us the Time from Midnight when the *Sun* rises; *m x*, which is call'd the *Amplitude* of the *Path* of the *Vertex* of *London* in the *Horizon* of the *Disc*; and *p m x* the *Sun's Amplitude* from the *East* at this time. All which are got by the following Proportions:

T. p m : *T. p x* :: *R* : *c.S. m p x*.
c.S. p x : *c.S. p m* :: *R* : *c.S. m x*.
S. p m : *R* :: *S. p x* : *S p m x*.

	T. p m:	38°.	28′.	00″.	9. 900086—
	T. p. ✶ : :	15.	32.	31.	19. 444221
R:	*c.S. m p* ✶.	69.	30.	34.	9. 544135
	15)	69.	30.	34.	(4ʰ, 38′ 02″
		60.	00.	00.	
		9.	30.	34.	

	c.S. p ✶:	15°.	32′.	31″.	9. 983822—
	c.S. p m : :	38.	28.	00.	19. 893745
R: :	*c.S. m* ✶.	35.	38.	27.	9. 909923

	S. p m:	38°.	28′.	00″.	9. 793831—
R: :	*S. p* ✶:	15.	32.	31.	19. 428043
	S. p m ✶.	25.	30.	51.	9. 634212

So that the Angle at the Pole which the *Proper Meridian* makes with the Hour-Circle of *Sun-Rising* is 69° 30′ 34″, which turn'd into Time is 4ʰ 38′ 02″; at which Time he rises to us at *London* when he is in this Point of the *Ecliptic*; and consequently he sets then at 7ʰ 21′ 58″; the Length of the Day being 14ʰ 43′ 56″; and that of the Night 9ʰ 16′ 04″. And from the *Second* Calculation it appears that the *Amplitude* of the *Path* of *London* in the *Horizon* of the *Disc* is at this Time, 35° 38′ 27″. From the *Third* Projection we find the *Sun*'s *Amplitude* from the *East*, 25° 30′ 51″; which in the Points of the *Compass* is *E. N. E.* 03° 00′ 51″, *Easterly*. Or the time of the *Sun*'s Rising, &c. and his *Azimuth* may be had in the Oblique-angl'd Triangle ☉ *p m*; in which ☉ *p m* is the *Hour-Angle* from *Sun*-Rising to *Noon*, and ☉ *m p* the *Azimuth* of the Rising *Sun* from the *North* Point of the *Horizon*: For in this Case we have three Sides, ☉ *p* the *Sun*'s Distance from the Pole, 74° 27′ 29″; *p m*, 38°

38° 28′ 00″; and *m* ☉ the *Sun*'s Distance from the *Vertex* when he rises, 90 Degrees. And for ☉ *p m* and ☉ *mp*, by the last Case save one of Oblique-angl'd *Spherical* Triangles, the Proportion is as follows:

R : *S. p m* : :	38°. 28′. 00″.	9. 7938317
S. p ☉ :	74. 27. 29.	9. 9838222
m ☉.	90. 00. 00.	
		9. 7776539
Sum.	202. 55. 29.	0. 2223461
S. ½ *Sum*	101. 27. 44½.	9. 9912506
S. Rem.	11. 27. 44½	9. 2982505
		19. 5118472
c.S. ½ *mp* ☉ ...	55°. 14′. 44″	9. 7559236
mp ☉ ..	110. 29. 28	
15°)	110. 29. 28.	(7^h 21′ 58″ *fere*
	105. 00. 00.	12. 00. 00.
	5. 29. 28.	4. 38. 02.

R : *S. p m* : :	38°. 28′. 00″.	9. 7938317
S. m ☉ :	90. 00. 00.	10. 0000000
p ☉.	74. 27. 29.	9. 7938317
Z.	202. 55. 29.	0. 2061683
S. ½ Z.	101. 27. 44½.	9. 9912506
S. X^ie	27. 00. 15½.	9. 6571107
		19. 8545296
c.S. ½ ☉ *mp.*	32°. 14′. 35″	9. 9272648
☉ *mp.*	64. 29. 10.	
	25. 30. 51. +	
	90. 00. 01.	

From which we find that according to this Calculation the *Sun* at Noon is 7^h 21′ 58″ from the Rising;

Rising; or, which is the same thing, rises at $4^h\ 38'\ 02''$ as before: And his Rising *Azimuth* from the *North* is $64^\circ\ 29'\ 10''$, making his *Amplitude* from the *East* $25^\circ\ 30'\ 50''$; which by the former *Calculus* was but one *Second* more; *viz.* $25^\circ\ 30'\ 51''$.

Q. E. I.

Scholium 1. After the same manner that we find out the Times when the *Sun* rises and sets, the Length of the Day, and the *Azimuths* which he rises and sets upon in this *Problem*, from his Place in the *Ecliptic* and Declination known; we may discover the Continuance of any *Fix'd Star* above the *Horizon*, its Time of Rising and Setting, and the *Azimuths* which it rises and sets upon, having its Right Ascension and Declination given. For the Difference of its Right Ascension, and that of the *Sun* converted into Time, tells us when it comes upon the *Meridian*; and having its Declination we have, what is equal to it, the *Reflection*; and therefore in a Right-angl'd Triangle, for instance, *p x m*, the Sides *p x*, *p m*, and the Angle at *x*; from whence we may easily find *x p m*, the Complement to 180 *Degrees* of half the Time it is above the *Horizon*; which taken from 180 leaves an Arch which turn'd into Time lets us know how many Hours before it comes to the *Meridian* it rises, or appears above the *Horizon*; and *p m x* its *Amplitude* from the *East*. And after the same manner may we know when, and upon what Point it sets.

Schol. 2. Besides the way of finding out the Latitude of the Place we are in, taken notice of in the *Third Scholium* to the preceding *Problem*, there are several other ways of determining it, either by *Calculation* or *Observation*, to be met with amongst the *Astronomical* Writers. To do it by *Observa-*

Observation is the best; and perhaps there is no way of investigating it by *Observation*, in Places of considerable Latitude, less liable to Exception than this following. Having an Arch of a Circle of *Five* or *Six* Foot *Radius*, of near 180 *Degrees*, Rightly graduated, and duely fix'd in the Plane of the *Meridian*, provided with *Telescopic* Sights and all other Requisites; if we *observe* the greatest Altitude of a *Fix'd Star* near the *North* Pole of the World when it comes upon the *Meridian*, in the beginning of a long Winter-Evening; and wait till after 12 Hours time it comes upon the *Meridian* again, and then *observe* its least *Meridional* Altitude, and correct both these Altitudes *observ'd* by making due Allowances for *Refractions*; half the Difference of these *Observ'd Meridional* Altitudes added to the lesser will give us the Elevation of the *North* Pole of the World above the *Horizon*; or (which is Equal to it) the Latitude of the Place where we make our Observation: Its Complement to 90 *Degrees*, being the Height of the *Equator* in that Latitude. The reason of which is plain to any one who has in the least dipt into *Astronomical* Matters: For as the *Sun* and *Fix'd Stars* seem to move round the *Earth* in 24 Hours time in Circles parallel to the *Equator*, and consequently which have the same Poles with it; by reason of the real Circumrotation of the *Earth* upon the *Axis* of the *Equator* in that Space of Time; consequently the *Observ'd Fix'd Star* must seem to move thro' its Parallel in that Time; and being so near the *North* Pole as in its whole Circuit to keep above the *Horizon*, it must be seen twice upon the *Meridian*, once so much above the Pole and the other time so much below it, as it is distant from it; half therefore of the Difference 'twixt these Two *Meridional* Altitudes is its Distance from the Pole, which added

to

to its least *Meridional* Altitude must give us the Height of the Pole itself. In the Choice of our *Stars* by which to make *Observations* of this kind we should always pitch upon such as are nearest the *Pole*, when they can be had; because they will be found the highest in the least *Meridional* Altitudes, and consequently our *Observations* will be less liable to be corrupted by *Refractions*. And whereas this method will not do with those who have not considerable either *North* or *South* Latitude, they must take that mention'd in the End of the Third *Scholium* to the preceding *Problem*: Or they may find out the Latitude of the Place they are in, by *observing* the *Meridional* Height of any *Fix'd Star* whose Declination is known. For if that *Star* has *Northern* Declination, and to the *Vertex of* the *Observer* comes upon the *South* Part of the *Meridian*, then its Declination *subtracted* from its *Meridional* Height leaves the Height of the *Equator*, which taken from 90 *Degrees* leaves the *North* Latitude of the Place: If it has *Southern* Declination, and appears on the same Part of the *Meridian*, then that Declination *added* to its *Meridional* Height gives the Height of the *Equator*; which if it is less than 90 *Degrees* its Complement to a *Quadrant* is the Latitude *North*; if more, the Excess above 90 *Degrees* is the Latitude *South*. If that *Star* has *Southern* Declination, and to the *Vertex* of the *Observer* comes upon the *North* Part of the *Meridian*, then its Declination *subtracted* from its *Meridional* Height leaves the Height of the *Equator*, which taken from 90 *Degrees* leaves the *South* Latitude of the Place: If it has *Northern*, and appears upon the same Part of the *Meridian*, then that Declination *added* to its *Meridional* Height gives the Height of the *Equator*, which if it is less than 90 *Degrees* its Complement to a *Quadrant* is

is the Latitude *South*; if more, the Excess above 90 *Degrees* is the Latitude *North*. And the same holds when the Latitudes of Places are taken from *Observ'd Meridional* Heights of the *Sun*. By the Height of the *Equator* when above 90 *Degrees* (as it can never be really above 90 *Degrees* elevated above the *Horizon*) the Reader must see that I mean its Distance from the *North* and *South* Points of the *Meridian* where that Circle intersects the *Horizon*.

PROBLEM II.

The Sun's *Distance from the Pole, the Latitude of the Place we are in, and the Time from* Noon *when we require his* Height *and* Azimuth *being given; to find out his Distance from the* Vertex, *and* Azimuth *at that time.*

SOLUTION *upon the Projection.*

LET the *Sun's* Place (and consequently his Distance from the Pole) and the Latitude of the Place we are in, be as in the preceding *Problem*; and let his *Azimuth* and Distance from the *Vertex* be requir'd at *Six* in the Morning: 'Tis plain that *d p* ☉ being the *Proper Meridian*, *s p g* at Right Angles to it must be the Hour-Circle of *Six* in the Morning and Evening; and the *Sun* rising when the *Vertex* comes at *m*, at *Six-a-Clock* in the Morning it must be at *u*; and if we strike a Circle thro' ☉, *u*, and *d*, it will be the *Vertical* Circle passing thro' the *Sun* at that time; *viz.* *d u* ☉, of which *u* ☉ is the Distance of the *Sun* from the *Vertex*: And having the Centre of this New Circle we may have its *Inclination* to the Plane of the Projection, by *Coroll. Prop.* III. *Sect.* II. *Cap.* II. and consequently its Pole which falls within that Plane, by *Lemma* I. *Sect.* III. *Fig.* 32.

Cap. II.

Cap. II. From whence we may discover the Value of *u* ☉, the *Sun's* Distance from the *Vertex* at this time: By *Coroll.* 1. *Lemma* II. *Sect.* III. *Cap.* II. And for the Angle *p u* ☉, which is the *Azimuth* of the *Sun* from the *North* at this Time, cutting off a projected Quadrant from *d u* ☉, from *u* thro' ☉ in that Circle continued as far as requisite, by *Coroll.* 1. *Lemma* II. *Sect.* III. *Cap.* II. and another from *g p f*, from *u* thro' *p* towards *f*, by the same: A Circle struck thro' those New Quadrantal Points, and a Third from *u* to the contrary Side in either of the Projected Circles *d u* ☉ or *f u g* (as all great Circles of the *Sphere* cut each other at a Semi-Circle's Distance) will be the Circle in which the Angle *p u* ☉ is to be measured; and so much of it as is intercepted 'twixt the Two Quadrantal Points first found will measure that Angle: And having the Centre of that Circle we can have its *Inclination* to the Plane of the Projection; and drawing a Line thro' that Centre and that of the Projection to its Circumference, we may find its Pole within the Plane of the Projection; and having that, we can discover how much the Arch of the New Circle intercepted 'twixt the Two First Quadrantal Points answers to in the *Ecliptic*: That is, the Value of the Angle *p u* ☉, the *Azimuth* of the *Sun* from the *North* at *Six* in the Morning.

Q. E. F.

Trigonometrical SOLUTION.

THIS *Problem* is with as little Trouble, and more Accuracy solv'd by *Trigonometrical Calculation*: For in the Triangle ☉ *p u* Right-angl'd at *p*, because the *Vertex* at *u* moves Six Hours in Time, or 90 *Degrees* (15 *Degrees* in the *Equator* answering to an Hour of Time) to come

to ☉ when it is upon the *Meridian*; having p ☉, and $p\,u$, we may easily find out u ☉ and $p\,u$ ☉, by the following Proportions:

$$S.\,up : R :: T.\,p☉ : T.\,pu☉.$$
$$R : c.S.\,p☉ :: c.S.\,up : c.S.\,u☉.$$

$S.\,up$:	38°.	28′.	00″.	9. 793831—
R :: $T.\,p$☉.	74.	27.	29.	10. 555772
$T.\,pu$☉.	80.	11.	05.	10. 761941

$R : c.S.\,p$☉ ::	74°.	27′.	29″.	9. 428044
$c.S.\,pu$:	38.	28.	00.	9. 893745
$c.S\,u$☉.	77.	53.	24.	9. 321789

So that the *Sun*'s Distance from the *Vertex* at this time is 77° 53′ 24″; and his *Azimuth* from the *North* 80° 11′ 05″: And consequently his Height is 12° 06′ 36″; and his *Amplitude* from the *East* 9° 48′ 55″; that is 1° 26′ 05″ less than one Point of the *Compass*, that being 11°. 15′ 00″.

Q. E. I.

Scholium. If it had been requir'd to discover at what time in the Morning, and at what Height of the *Sun*, he is upon the *Prime Vertical*, or *due East* Point; then, a Great Circle passing thro' the *Vertex* and the *Pole*, suppose $u\,p$, and another thro' the *Vertex* and the *Sun*, suppose u ☉, cutting each other at Right-Angles in the Point u, (as the *North* and *East* Points are at a Quadrantal Distance from each other in the *Horizon*, of which the *Vertex* of our Place is the Pole) we may easily find in the Triangle $u\,p$ ☉ supposed Right-angl'd at u, the Angle $u\,p$ ☉ the time before *Noon* when the *Sun* is *due East*; and the Side

Side u ☉ his Distance from the *Vertex*, the Complement of which to 90 *Degrees* is his Altitude at that time, by the following Proportions:

$T.p$☉ : $T.pu$: : R : $c.S.up$☉.
$c.S.pu$: R : : $c.S.p$☉ : $c.S.u$☉.

$T.p$☉ : 74°. 27'. 29". 10. 555772—
$T.pu$: : 38. 28. 00. 19. 900086
R : $c.S.up$☉. 77. 14. 05. 9. 344294
15) 77. 14. 05. (5^h. 08'. 56" $\frac{1}{3}$
75. 12. 00. 00.
2. 14. 05. 6. 51. 03 $\frac{2}{3}$

$c.S.pu$: 38°. 28'. 00". 9. 893745—
R : : $c.S.p$☉ : 74. 27. 29. 19. 428044
$c.S.u$☉. 69. 59. 18. 9. 534299
90. 00. 00.
20. 00. 42.

So that the *Sun* is *due East* at 6^h 51' 03" $\frac{2}{3}$ in the Morning; being distant from the *Vertex* 69° 59' 18", and consequently 20° 00' 42" high. Or these may be found out upon the Projection by any one who understands the Solution upon the Projection of the *Problem* to which this *Scholium* is annex'd: Andin like manner may we find out the *Sun's* *Azimuth* and Distance from the *Vertex* to any time assignable; or at what Time and Height he will appear upon any assignable *Azimuth*. After the same manner also, having the Right Ascension and Declination of a *Fix'd Star*, we may answer any of these Questions with relation to it: For having the Difference of Right Ascension 'twixt it and the *Sun*, that converted into Time tells us when it shall come upon the *Meridian*, and knowing that, and its Declination,

clination, and Distance from the *Meridian*; we may easily find its *Azimuth* and Distance from the *Vertex*: Or if it has Eastern Amplitude at its Rising, we may easily discover at what Time, and Distance from the *Vertex*, it is *due East*.

PROBLEM III.

Having the Sun's *Declination from the Equator, the Latitude of the Place we are in, and his Distance from the* Vertex *by Observation; to find out the Time of the Day when that Observation was made, and the* Azimuth *the* Sun *was then upon.*

SOLUTION *upon the Projection.*

LET the *Sun's* Observ'd Distance from the *Vertex* be 46° 10′ 40″; and the former Data as before: *First*, for the Projecton of the Triangle upon which this *Problem* is solv'd; we have the Side *p* ⊙ already, and we want to project *v p* and *v* ⊙: For *v* ⊙ if we continue *d e* ⊙ as far as requisite for a Line of Measures in which to find the Centre of a lesser Circle of the *Sphere* Parallel to the *Horizon* of the *Disc* represented by *f e g*, or *Perpendicular* to the Plane of the Projection, distant from ⊙ its Pole 46° 10′ 40″; and project that Circle according to the Directions of *Prop.* II. *Sect.* II. *Cap.* II. it will cut the Path of the *Vertex* in the Point *v* thro' which the Circle *d u* ⊙ passes making the Arch *v* ⊙ represent 46° 10′ 40″ of that Circle when drawn thro' those *Three* Points; as all Parallel Circles cut off Arches of the Great Circles which pass thro' their Poles Equal to their own Distances from those Poles, and consequently their Representatives must do so by the Representatives of those Great Circles: And for the Side *p v* we *Fig.* 32.

have

have the Points p and v, and the projected *South* Pole of the World by which to project it. And, *Secondly*, having projected the Triangle; we may find out the *Azimuth* from the *North* $p\ v$ ⊙, and the Angle $v\ p$ ⊙ the Time from *Noon* when the *Sun* is 46° 10′ 40″ from the *Vertex* after the same manner that we find out $p\ v$ ⊙ in the Solution of the preceding *Problem*.

N. B. I do not complete this and some other Triangles, because it wou'd occasion Confusion in the Projection; but the Reader from what is said, may easily see how it must be done; and either supply it in his own Mind, or draw what Circles are to be supply'd in a separate Projection upon the same Plane. *Q. E. F.*

Trigonometrical SOLUTION.

BY *Trigonometrical Calculus*, in the Triangle $p\ v$ ⊙, having the Three Sides we may easily find the Angle $v\ p$ ⊙, the Time from *Noon* when the *Sun* is 46° 10′ 40″ from the *Vertex*, and $p\ v$ ⊙ his *Azimuth* from the *North* at that time, *viz.* by the *Eleventh* Case of Oblique-angled *Spherical* Triangles: In which we make use of the following Proportions:

R : *S.* $p\ v$ =	38°. 28′. 00″.	9. 7938317
S. p ⊙ :	74. 27. 29.	9. 9838221
v ⊙ :	46. 10. 40.	9. 7776538
Z.	159. 06. 09.	0. 2223462
S. ½ *Z.*	79. 33. 04. ½	9. 9927378
S. $X^{ia.}$	33. 22. 24. ½	9. 7404368
		19. 9555208
c.S. ½ $v\ p$ ⊙.	18°. 10′. 49″.	9. 9777604
$v\ p$ ⊙.	36. 21. 38.	
15°)	36. 21. 38.	(2h. 25′. 26″—
	30. 00. 00.	12. 00. 00.
	6. 21. 38.	9. 34. 34.

R : *S.*

R : S. p v =	38°. 28′. 00″.	9. 7938317
S. v ☉ :	46. 10. 40.	9. 8582313
p ☉.	74. 27. 29.	9. 6520630
Z.	159. 06. 09.	0. 3479370
S. $\frac{1}{2}$ Z.	79. 33. 04$\frac{1}{2}$.	9. 9927378
S. X^{ie}.	5. 05. 35$\frac{1}{2}$.	8. 9482954
		19. 2889702
c.S. $\frac{1}{2}$ p v ☉.	63°. 49′. 46″.	9. 6444851
p v ☉.	127. 39. 32.	
11° $\frac{1}{4}$)	127. 39. 33. (11 P^{ts}. . . . +	
	03°. 54′. 32″.	

So that the *Sun* is 46° 10′ 40″ from the *Vertex*, or 43° 49′ 20″ in a *Vertical* Circle above the *Horizon* at 9^h 34′ 34″ in the Morning being then 11 Points of the Compaſs, and 03° 54′ 32″ from the *North*, counting round by the *Eaſt*. Or he is at the ſame Height, or Diſtance from the *Vertex* (if the Obſervation is made in the Afternoon) at 2^h 25′ 26″ in the Afternoon; being then 11 Points of the Compaſs, and 03° 54′ 32″ from the *North*, counting round by the *Weſt*: Or, in the former Caſe *S. E.* by *E.* 03° 54′ 32″ *Southerly*: In the Latter, *S. W.* by *W.* and 03° 54′ 32″ *Southerly*.

Q. E. I.

Schol. 1. From this *Problem* we are inabled to regulate any *Movement* by the *Sun* at any time of the Day, without ſtaying till he comes to the *Meridian*. For obſerving by a large *Quadrant* his Height at any time; and correcting that *Viſible* Height by a due Allowance for his *Parallax* and *Refractions*; and noting the *Hour*, *Minute*, and *Second* when our *Obſervation* was made, upon the *Movement* we are about to regulate; if we calculate the true Time of our *Obſervation* from

K 2 that

that Height, the Latitude of the Place we are in, and the *Sun*'s Declination given, or rather the Complements of those Three to 90 *Degrees*; the Difference 'twixt that and the Time pointed out by our *Movement* at the Moment of Observation, will shew us how much our *Clock* is before or behind the *Solar* or Apparent Time: Which is necessary to be known in *Astronomical Observations* of all Kinds; and without which we cannot compare *observ'd* with *predicted Eclipses*, *Planets-Places*, &c. in order to Examine the Orbits, Periodical Times, *&c.* already assign'd 'em by the *Astronomers*, and to correct 'em if they are found in any respect amiss.

Schol. 2. With the same *Data* we may also derive the Time of the *Night* from any observ'd *Fix'd Star* whose Right Ascension and Declination are known; together with the *Azimuth* that *Star* is then upon. For *observing* its Distance from the *Vertex* at the Time, and having from its Declination its Distance from the *North* Pole, and the Complement of the Latitude of the Place we are in, we have Three Sides of such a Triangle as *p v* ☉, and consequently may find the Angle *p u* ☉ the *Star*'s *Azimuth* from the *North* at the Time of *Observation*, and *u p* ☉ which turn'd into Time tells us how long it will be ere this *Star* comes upon the *Meridian*, if it is on the *Eastern* Side of it, or how long it has pass'd it if on the *Western*. And having the Right Ascension of the *Sun* at this time if it is *Less* than that of the *Star*, then *subtracting* it from the *Star*'s Right Ascension the *Residue* turn'd into Time tells us at what Distance from *Noon* the *Star* comes upon the *Meridian*: From which the Angle found turn'd into Time also *subtracted*, if the *Star* is on the *East* Side of the *Meridian*, the *Residue* gives us the true Time when we *observ'd* the *Star*'s Altitude; or *adding* it if the *Star*

Star is on the *Western* Side, the *Sum* gives us the Time of *Observation*. If the Right Ascension of the *Sun* is greater than that of the *Fix'd Star*, then to the *Star*'s Right Ascension must be added 360°, and from that Sum taking the *Star*'s Right Ascension, the Remainder turn'd into Time tells us the Time from *Noon* when the *Star* comes upon the *Meridian*, and the Angle found taken from, or added to, that Time (accordingly as the *Star* is on the *Eastern* or *Western* Side of the *Meridian*) gives us the Time of *Observation*. And whereas we cannot have the Right Ascension of the *Sun* from any Tables precisely to the Time of *Observation*, because that Time is suppos'd unknown, and what we are investigating; if we take it to the preceding *Noon*, having his Place in the *Ecliptic* at that time, and by the Additions of his Horary Motions to that Place for as many Hours as are pass'd 'twixt *Noon* and the Time of *Observation*, which our Movements will tell us with sufficient Accuracy for our present purpose, finding out his Place in the *Ecliptic* at the Time of *Observation*, we may thence deduce the Right Ascension of the *Sun* to the Time of *Observation* by the Process in the *First Problem* in the preceding *Section*. And thus finding out the Time of the *Night*, we may rectify our *Clocks* for any manner of *Astronomical Observations*; as those of the *Planets* Transits over the *Meridian*, or observ'd Distances from any *Fix'd Stars*, the Occultations of any *Fix'd Stars* by the *Moon*, *Lunar* Eclipses, Eclipses of the *Statellites* of *Jupiter*, and the like. We may also find out the Difference of Right Ascension of the *Sun* and any *Fix'd Star* by which we design to regulate our Movements, by setting an *Equatorial Clock* to the Point of *Twelve*, when the *Sun*'s Centre is observ'd to pass the *Meridian*; and the Time elaps'd 'twixt its Transit over the *Meridian*, and that of the *Fix'd Star* by which we

 make

make our *Observation* will give us their Difference of Right Aſcenſion; Or the ſame may be done by a *Common Clock*, making due Allowance for the Difference 'twixt *Solar* and *Equatorial* Hours for that Time, which is had from Tables fram'd by the *Aſtronomers* for that purpoſe.

Schol. 3. As we have occaſion, in deducing Right Aſcenſions of the *Fix'd Stars* or *Planets*, from comparing their *Tranſits* over the *Meridian* one with another or with that of the *Sun*, to turn *Solar* into *Equatorial* Time; the Difference of Right Aſcenſion 'twixt any Two *Phænomena* being always meaſur'd by an Arch of the *Equator*, intercepted 'twixt Two Great Circles paſſing thro' thoſe *Phænomena* and the two Poles of the World, and that Arch being diſcoverable by the *Equatorial* Time which paſſes 'twixt the Two *Tranſits*, and the *Equatorial* Time being generally to be deduc'd from the *Solar* according to which our Movements are moſt frequently kept; it may not be amiſs to give ſome Account here of the Difference 'twixt *Solar* and *Equatorial* Hours, and to ſhew how from the one we may deduce the other on any Day of the Year, or in any Point of the *Earth*'s Orbit. Now an *Equatorial* Day is that Space of Time which the *Earth* takes up in one complete Revolution upon its own *Axis*: and a *Solar* Day is that Space of Time which paſſes 'twixt the Moment when the Centre of the *Sun* leaves our *Meridian*, and that at which it comes upon it again. And whereas, by reaſon of the *Earth*'s *Annual* Motion, the *Sun* ſeems from Day to Day to move forward in the *Ecliptic* according to the *Series* of the *Signs*, nearly One *Degree*, but ſometimes more ſometimes leſs, according to the Place he is in in the *Ecliptic*; 'tis plain that a *Solar* muſt be ſo much longer than an *Equatorial* Day at any time aſſignable, as is the Arch of the

Equator

Equator which the *Earth* in its Circumrotation upon its own *Axis* must run, after a full *Equatorial* Revolution, to overtake the *Sun* in that Day's *Annual* Motion. Thus (*Fig* 32.) let the *Sun* be at *x* upon our *Meridian*, *p x* ♄ being the proper *Meridian* to this Place of the *Sun*, and ♄ that Point of the *Equator* which is upon the *Meridian* this Day at *Solar* Noon: When the *Earth* has made one complete Revolution upon its own *Axis*, so that the Point ♄ is upon this *Meridian* again, there has pass'd a Complete *Equatorial* Day; but during this Space of Time the *Sun* has seem'd to move in its Annual Orbit, according to the *Series* of the *Signs*, from *x* (for Instance) to ⊙; so that before he comes upon the *Meridian*, or a *Solar* Day is completed, the Point ♄ in the *Equator* must move on from ♄ to *æ*; or the *Meridian* *p x* ♄ must become (or come into the Place of) *p* ⊙ *æ*: The Portion therefore of the *Equator* ♄ *æ* (which is apparently nothing else but the Difference of the *Sun*'s Right Ascension this Day and the next at Noon, and may be had from his Place in the *Ecliptic* at both times, according to the Process of *Prob.* I. *Sect.* I. 9.) turn'd into Time tells us how much the *Solar* exceeds the *Equatorial* Day to this Place of the *Sun*: And if we add that Excess to 24 *Equatorial* Hours, and divide the *Sum* by 24, the *Quotient* will tell us how much a *Solar* exceeds an *Equatorial* Hour at this Time: And then, knowing the Difference of *Solar* Time 'twixt the *Transits* of the *Sun* (for Instance) and any *Fix'd Star* over the *Meridian*, we may easily reduce it to *Equatorial* Time; and that turn'd into *Degrees*, *Minutes* and *Seconds*, will give us their Difference of Right Ascensions. And whereas the *Earth*, in different Parts of its *Annual Orbit*, moves in one Day thro' different Portions of the *Ecliptic*; sometimes more, sometimes less,

accordingly as it is nearer to or more remote from its *Perihelion* Point, and its Motion is no Two Days precisely the same, but when it is in Places equally remote from that or the *Aphelion*; and the *Sun* must seem to do the same; Whereas, farther, the Arches of Right Ascension in the *Equator* answering to Arches of the *Earth*'s Annual Motion in the *Ecliptic*, are different in different Parts of the *Earth*'s Orbit: Yet still it always holds that the Difference 'twixt the Right Ascension of the *Sun* on one Day at Noon, and of the same at the next Noon, turn'd into Time, gives us the Excess of the *Solar* above the *Equatorial* Day at that time; wherever the *Sun* be, or at whatever time of the Year; as is plain from the Instance I have now given. And because of these Varieties the *Astronomers* Calculate the *Sun*'s Right Ascensions to every Day in the Year, and thence deduce his Ascensional Differences from Day to Day, and by those Differences turn'd into Time adjust the Excess of the *Solar* above the *Equatorial* Day all the Year round; and putting these in *Tables*, make use of 'em for the *Reduction* of *Solar* to *Equatorial* Time, whenever they have occasion for it in *observing* the *Transits* of the *Fix'd Stars* or *Planets* over the *Meridian*; and deducing the Right Ascensions of the *Planets* from the *Fix'd Stars*, or of one *Fix'd Star* from another, or of either *Fix'd Star* or *Planet* from the *Sun*: And thus knowing the Difference of Right Ascensions 'twixt the *Sun* at a given Noon and any *Fix'd Star* assignable, we can tell when that Star will come upon the *Meridian* in *Equatorial* Time, and converting that into *Solar*, at what *Solar* Time of the Night it will be upon the *Meridian*; and knowing that, and *observing* its Height at any time that Evening, and having its Declination, we may thence easily deduce the Time of the Night

when

when we made our *Observation*, by the Procefs in the prefent *Problem*.

Schol. 4. The *Mean* Time, to which the *Solar Tables* and thofe of all the *Planets* are Calculated, is different both from the *Solar* and *Equatorial*: For whereas a *Solar* Day is that Space of Time which paffes 'twixt the Moment when the Centre of the *Sun* leaves our *Meridian* and that at which it comes upon it again; which is meafured by the whole *Equator*, and an Additional Arch of the fame Equal to the Afcenfional Difference 'twixt the Place in the *Ecliptic* in which the *Sun* leaves, and that in which he comes again upon our *Meridian*; which Arch is of different Lengths in different Parts of the *Earth*'s Orbit, whence arifes an Inequality in *Solar* Days: And whereas, tho' *Equatorial* Days (being always meafur'd out by one Compleat Revolution of the *Equator*) are all Equal to one another, for any thing the moft acute either *Aftronomers*, *Philofophers* or *Geometricians* have been able to difcover to the contrary; yet they do not exactly Anfwer to the *Mean* Diurnal *Apparent* Motion of the *Sun*; but 366 *Equatorial* Days are pafs'd whilft he compleats his Annual *Mean* Motions: Whereas *Laftly*, the *Mean* Motions of the *Sun* and *Planets* are found Equable and Uniform, as they pafs over Equal *Areas* of their *Orbs* in Equal Times; and the Periodical Times and *Mean* Motions of the reft of the *Planets* are adjufted to and compar'd with the *Apparent* Periodical Time and *Mean* Motions of the *Sun*: Hence the *Aftronomers* have been oblig'd, in making *Tables* reprefenting the *Mean* Motions of the *Sun* and *Planets*, to adapt them to what they call a *Mean* Time, which flows on Uniformly as their *Mean* Motions do; that as from their *Mean* Motions they may have their Places in the Heavens, fo from the *Mean* Time apper-

appertaining to those Motions they may find out the *Apparent Solar* Time in which they are in those Places. Now the Apparent Periodical Time and *Mean* Motions of the *Sun* being those by which those of the *Planets* are adjusted, and with which they are compar'd, the *Mean* Time in which he performs any of his *Mean* Motions is made the Measure of theirs, as it not only shews us Uniformly the *Mean* Motions of the *Sun*, but of all the *Planets*, from whatever *Epocha* or *Period* we begin to compare 'em. And this *Mean* Time Answering to the *Mean* Motions of the *Sun*, and to which both his and those of all the *Planets* are Calculated, is got after the following manner. If the *Sun* seems to move round the whole *Area* of the *Earth*'s Orbit, or to complete one intire Revolution about the *Earth*, in $365\frac{1}{4}$ *Solar* Days nearly; then as every *Solar* Day requires one Whole Revolution of the *Equator* and something more, which is more or less according to the Part of his Orbit the *Sun* is in; so the whole *Solar* Year is mark'd out by as many Revolutions of the *Equator* as there are Days in it, and one Whole Revolution more made up of the Additional Parts, got by each Day of that Year: So that whereas there are but $365\frac{1}{4}$ *Solar* Days in a *Solar* Year, these are measured out by 366 Revolutions of the *Equator*; and consequently 366 *Equatorial* Days are Equal to the Time in which the *Sun* completes his Annual *Mean* Motions, or to $365\frac{1}{4}$ *Solar* Days; the Former of which therefore divided by the Latter shews us that if an *Equatorial* Day is 24 Hours, a Day in which the *Sun* in his *Mean* Motions runs $\frac{1}{365}$, *&c.* of his Orbit is $24^h\ 03'\ 56''\frac{1}{2}$ nearly; or, $59'\ 08''$ of a *Degree* being the *Sun*'s *Mean* Diurnal Motion, he runs it in $24^h\ 03'\ 56''\frac{1}{2}$ of *Equatorial* Time, or a *Mean* Day. To find out therefore the *Mean* Time Answering

ſwering to the *Mean* Motions of the *Sun*; or its Exceſs above, or how much it comes ſhort of the *Apparent Solar* Time, which is made by the *Earth*'s True Motion (ſeeming to us to belong to the *Sun*) from time to time; beginning with whatever Point in the *Ecliptic* he is in upon the *Meridian* at any aſſignable Noon (as he will have run 59′ 08″ of his *Mean* Motions in one *Mean* Day, 1° 58′ 16″ in the *Second*, and ſo on; till in a Year he completes his Annual *Mean* Motions, and returns to that Point of the *Ecliptic* where we begun with him) If we Calculate what Arch of the *Ecliptic* Anſwers to 59′ 08″ of his *Mean* Motions for the Firſt *Mean* Day, after he leaves this Point of the *Ecliptic*, and ſo on round his whole Orbit; if his *Mean* Motion in the firſt *Mean* Day is greater than the True Arch in the *Ecliptic* which he ſeems to run, then the Right Aſcenſion of the Latter taken from that of the Former leaves an Arch, which turn'd into Time tells us how much the *Solar* is leſs than the *Mean* Day, and conſequently how much the *Solar* comes before the *Mean* Noon; which *ſubtracted* therefore from the *Solar* leaves the *Mean* Time: If the *Mean* Motion in this Day is leſs than the Arch which he runs in the *Ecliptic*, then the Right Aſcenſion of the Former taken from that of the Latter leaves an Arch, which turn'd into Time tells us how much the *Mean* is leſs than the *Solar* Day, and conſequently how much the *Mean* comes before the *Solar* Noon; which *added* therefore to the *Solar* gives the *Mean* Time. And after the ſame way we get theſe *Proſthaphæretical* Arches all the Year round; with their *Titles*, whether they are to be *added* to, or *ſubtracted* from the *Apparent* to give the *Mean* Time: And putting the Time Anſwering to 'em into *Tables* in order, from the *Firſt* Point of *Aries* thro' the whole *Ecliptic*; and annexing their *Titles*,

tles, whether they are to be *added* to, or *subtracted* from the *Apparent* in order to have the *Mean* Time; we have what the *Astronomers* call *Tables of the Equations of Natural Days*; from whence having the *Apparent* we may discover the *Mean* Time to every Day of the Year, or of as many Years as we Calculate these *Equation-Tables* to. Or, having the *Mean* Time we may find the *Apparent* if we *add* where these *Tables* direct us to *subtract*, and *subtract* where they order us to *add*, the *Equation* annex'd to any *Degree* or *Minute* of the *Ecliptic*, according to the Time of the Year when we want to reduce the *Mean* into *Apparent* Time. And to the *Mean* Time got after the foregoing manner the *Tables* of the *Mean* Motions of all the *Planets* are Calculated, and from them and the *Tables* of the *Equations* of their Orbits, &c. their true Places are got to any Point of *Mean* Time past or to come; and knowing the *Mean* we may easily find out the *Apparent* Time when they are in those Places. And thus if we have a mind to try any *Tables* of the Motions of the *Planets* by *Observations*, if we have the *Apparent* Time of the *Observ'd Geocentrical* Place of a *Planet*, we may know in what Point of *Mean* Time it is in that Place; and if we Calculate its *Heliocentrical*, and thence deduce its *Geocentrical* Place, from the *Tables*, to that point of *Mean* Time, and find the *Tables* give us the *Observ'd* Place, we must so far conclude 'em good: If they give us a different Place, then they must be look'd upon as Faulty in Proportion to that Difference. But this in some measure by the Bye; as it has not a direct Relation to what more particularly belongs to this Place, *viz.* the *Solution* of the Diurnal *Phænomena* upon the *Projection* of the *Sphere* on the Plane of any one of its Great Circles.

PROBLEM

Problem IV.

Having the Latitude of the Place we are in, the Time of the Day when we make our Observation, and the Sun's *Place and Right Ascension at that Time; to find out what Point of the* Ecliptic *culminates upon the* Meridian; *what is the highest Point of it, or the* 90th Degree *from the Points where it intersects the* Horizon, *and consequently those Points themselves; the Distance of the* Nonagesimal *and Mid-Heaven Points from the* Vertex; *and the Angle which the* Vertical *Circle passing through the* Sun *at that Time makes with the* Ecliptic.

Solution *upon the Projection.*

LET the *Sun's* Place and Right Ascension be as in the Projection, and let these Particulars be investigated to *One a-Clock* in the *Afternoon*: As *c* in the Tangent-Line *a b c* is the Centre of the proper *Meridian* to this Place of the *Sun*, *b c* being the Tangent of its Right Ascension, 39° 48′ 05″, to the *Radius b p*; if we add to this 15 *Degrees*, making it 54° 48′ 05″, we shall find *b* ♑ the Tangent of the Right Ascension, of the *Mid-Heaven* at *One-a-Clock*; *p W C* the *Meridian* at that Time, *W* being the Point in the *Path* where the *Vertex* then is, *C* that Point of the *Ecliptic* which then culminates upon the *Meridian*, and *W* ☉ ♋ the Angle which the *Vertical* Circle passing thro' the *Sun* at that Time makes with the *Ecliptic*. And if we draw a Right Line from *e* thro' *W* to the *Ecliptic* it will cut in the Point *N*, the Point of it which is evidently nearest the *Vertex* or the highest or *Nonagesime* Point of it; *N W* being the Distance of that Point from the *Vertex*, and *W C* Fig. 32.

WC being the Distance of the same from that Point of the *Ecliptic* which culminates upon the *Meridian*. So that in the Latitude of *London*, to this Time of the Day, and Place of the *Sun*, we want *C* the Point of the *Ecliptic* which culminates upon the *Meridian*, *N* the *Nonagesimal* or Highest Point of it; *WN* the Distance of the *Nonagesimal* Point, and *WC* that of the Point then culminating upon the *Meridian*, from the *Vertex*; and *W*☉ ♋ the Angle which the *Vertical* Circle passing thro' the *Sun* makes with the *Ecliptic* at this time. For which we may proceed as follows:

If we open the *Sector* to the *Radius* of our Projection, and take ♈ *C* 'twixt our Compasses and measure it upon the *Line of Chords*, we shall find how far the Point *C* which then culminates upon the *Meridian* is distant from the *First* Point of *Aries*, and consequently that Point itself; and after the same manner may we discover the *Nonagesime* at *N*, and consequently the Points of the *Ecliptic* where it intersects the *Horizon*; and subtracting the lesser of these from the greater, we have the Distance of the *Nonagesime* from the *Mid-Heaven*: For the Distance of the *Nonagesime* from the *Vertex*, if we draw a Line at Right Angles to *eWN* thro' the Point *e* till it intersects the *Ecliptic* on both Sides, and lay a Ruler on that Point of Intersection which falls 'twixt *g* and ♈ continuing it thro' *W* to the *Ecliptic*, the Arch intercepted 'twixt the Points where that Ruler intersects the *Ecliptic* and the Point of the Line drawn at Right Angles to *eWN*, where it intersects the *Ecliptic* 'twixt *f* and ♎ measured upon the *Line of Chords* gives us the Value of *eW*, by *Coroll. Prop.* I. *Sect.* III. *Cap.* II. which taken from 90 *Degrees* leaves *WN* the Distance of the *Nonagesime* from the *Vertex* at this time: For *WC*, having the

the Centre of pW♋ at ♑ we have its *Inclination* to the Plane of the Projection, by *Prop.* III. *Sect.* II. *Cap.* II. and consequently may have its Pole within that Plane, by *Lemma* I. *Sect.* III. *Cap.* II. and the Value of *p C*, by *Coroll.* II. *Lemma* II. *Sect.* III. *Chap.* II. from which taking *p W* the Complement of the Latitude of the Place we are in, we have *WC* the Distance of that Point of the *Ecliptic* from the *Vertex* which culminates upon the *Meridian*. And for the Angle *W* ☉ ♋, if we take a Quadrant of the *Ecliptic* from ☉ towards ♎, and another from the Circle ☉ *W d*, (drawn thro' the *Sun*, the *Vertex* and the Point in the *Ecliptic* opposite to that where the *Sun* is) from ☉ towards *d*, according to the Directions of *Coroll.* 1. *Lemma* II. *Sect.* III. *Cap.* II. and thro' the *Quadrantal* Points found, and the Point opposite to that found in the *Ecliptic*, strike a Great Circle; having its Centre we may have its *Inclination* to the Plane of the Projection, as in the Case of *p WC*, and its Pole within that Plane; and having that Pole we may measure the Arch intercepted 'twixt the Two *Quadrantal* Points found, which gives us the Value of the Angle which the *Vertical* Circle passing thro' the *Sun* makes with the *Ecliptic* at this Time, *viz.* *W* ☉ ♋.

Q. E. F.

Trigonometrical SOLUTION.

FOR the *Trigonometrical* Determination of these Particulars, *First*, in the Right-angl'd Triangle *p* ♋ *C*, having *p* ♋ the Distance of the *Tropic* of *Cancer* from the Pole of the World, 66° 31′ 00″; ♋ *p C* the Complement of *C p* ♈ to 90 *Degrees*, 35° 11′ 55″; and the Angle at ♋ Right;

Right; we may find out ♋ C and Cp by the following Proportions:

R : S. p ♋ : : *T. Cp* ♋ : *T. C* ♋.
c.S.Cp ♋ : *R* : : *T. p* ♋ : *T. p C.*

R : *S. p* ♋ : :	66°. 31′. 00″.	9. 962452
T. Cp ♋ :	35. 11. 55.	9. 848426
T. C ♋.	32. 54. 05.	9. 810878

c.S. Cp ♋ :	35°. 11′. 55″.	9. 912307—
R : : *T. p* ♋ :	66. 31. 00.	20. 362043
T. p C.	70. 27. 13	10. 449736

So that *C* ♋ is 32° 54′ 05″, which taken from ♋ ♈, 90 *Degrees*, leaves ♈ *C* 57° 05′ 55″; and consequently *C* that Point of the *Ecliptic* which culminates upon the *Meridian* at this time is in ♉, 27° 05′ 55″; From whence we may easily discover what Point of the *Ecliptic* rises at this time, and what Point sets: And *p C* is 70° 27′ 13″, from which deducting the Complement of the Latitude of the Place we are in, we have *W C* the Distance of that Point of the *Ecliptic* which now culminates upon the *Meridian* from the *Vertex*, 31° 59′ 13″: The Complement of which to 90 *Degrees* is the Height of the *Ecliptic* at this time; Or, the *Inclination* of the *Ecliptical* to the *Horizontal* Plane.

Secondly, In the Oblique-angl'd Triangle *e p W*, Obtuse-angl'd at *W*; having *e p* the Distance of the Two Poles of the *Equator* and the *Ecliptic*, 23° 29′ 00″; *p W* the Semidiameter of the *Path* of our *Vertex*, 38° 28′ 00″; and *e p W* the Complement of the Distance of the *Mid*-Heaven from the *First* Point of *Cancer* to 180 *Degrees*, 144° 48′ 05″: We may have *e W*, which taken from 90 *Degrees* leaves

leaves *WN* the Distance of the *Nonagesime* from the *Vertex*; and *peW* the Difference of Longitude 'twixt the *Nonagesime* and the *First* Point of *Cancer*. For the Obtuse Angle at *p* being given, if we let fall a Perpendicular from *W* to *R*; in the Triangle *RpW* having *Wp*, *WpR*, and the Angle at *R* Right; we may have *pR*, and *RW* by the following Proportions:

$$R : c.S.\ WpR :: T.\ Wp : T.\ pR.$$
$$R : S.\ WpR :: S.\ Wp : S.\ WR.$$

$R : c.S.\ WpR ::$	35°. 11'. 55".	9. 912306
$T.\ Wp :$	38. 28. 00.	9. 900086
$T.\ pR.$	32. 59. 33.	9. 812392

$R : S.$	9. 760732
$S.$	9. 793831
$S.$	9. 554563 = *WR*. 21°. 00'. 43".

So that *pR* being 32° 59' 33", if we add to it *ep*, 23° 29' 00", because the Perpendicular falls without the Triangle, *eR* will be 56° 28' 33": And in the *Triangle eRW* having *eR*, 56° 28' 33'; *WR* 21° 00' 43"; and the Angle at *R* Right; we may have *eW* and *ReW* by the following Proportions:

$$S.\ eR : R :: T.\ RW : T.\ ReW.$$
$$R : c.S.\ eR :: c.S.\ RW : c.S.\ eW.$$

$B\ eR :$	56°. 28'. 33".	9. 920984 —
$R :: T.\ RW :$	21. 00. 43.	19. 584447
$T.\ ReW.$	24. 44. 16.	9. 663463

$R : c.S.\ eR :$	56°. 28'. 33".	9. 742166
$c.S.\ WR :$	21. 00. 43.	9. 970117
$c.S.\ eW$	58°. 57'. 53".	9. 712283

So that *R e W* being 24° 44′ 16″ Equal to ♋ *e N*, if we take it from 90 *Degrees* it will leave ♈ *e N* or ♈ *N* the Distance of the *Nonagesime* from the *First* Point of *Aries*, 65° 15′ 44″; and therefore the *Nonagesimal* Point of the *Ecliptic* at this time in ♊, 05° 15′ 44″. And *e W* being 58° 57′ 53″, if we take it from 90 *Degrees*, it will leave *W N* the Distance of the *Nonagesime* from the *Vertex* 31° 02′ 07″; 58° 57′ 53″ being the Altitude of the *Ecliptic* at this Time above the *Horizon*; or the Angle which the Planes of the *Ecliptic* and the *Horizon* make with each other. *Virgo* is 05° 15′ 44″ Rising at this time, and *Pisces* 05° 15′ 44″ being that Point of the *Ecliptic* which then Sets. And the Distance of the *Nonagesime* from the *Mid-Heaven* is 08° 09′ 49″.

Thirdly, For the Angle *W* ☉ *N*, or that which the *Vertical* Circle passing thro' the *Sun* makes with the *Ecliptic* at this time, call'd the *Parallactic* Angle; in the Triangle *N W* ☉ we have *N W*, the Distance of the *Nonagesime* from the *Vertex*, 31° 02′ 07″; ☉ *N* the Distance of the *Nonagesime* from the *Sun*, 23° 00′ 31″; and the Angle at *N* Right; and consequently may easily find out *W* ☉ *N*, by the following *Analogy*:

S. ☉ *N* : *R* :: *T. N W* : *T. N* ☉ *W*.

S. ☉ *N*:	23°.00′.31″.	9. 592032—
R :: *T. N W*:	31. 02. 07.	19. 779378
T. N ☉ *W*.	56. 59. 30.	10. 187346

So

So that the Parallactical Angle at this time is	56°	59′	30″
That Point of the *Ecliptic* which culminates on the *Meridian* being ♉	27	05	55
Its Distance from the *Vertex*	31	59	13
The Highest or *Nonagesime* Point of the *Ecliptic* ♊	05	15	44
Its Distance from the *Vertex*	31	02	07
The Rising Point of the *Ecliptic* ♍	05	15	44
The Setting Point ♓	05	15	44
The Distance of the *Nonagesime* from the *Mid-Heaven*	08	09	49

Q. E. I.

Scholium. If the *Parallactical* Angle is to be investigated at the *Moon* (or any of the other *Planets* which make a Sensible *Parallax* with the Semidiameter of the *Earth*) at the same time; and she is wide of the *Ecliptic* (for Instance) towards the *North*; then knowing her Longitude and Latitude, in the Triangle *We* ☽ we have *We* the Distance of the *Vertex* from the Pole of the *Ecliptic* at this time, *e* ☽ the Complement of the *Moon*'s Latitude, and *We* ☽ = *Ne S* the Difference of Longitude of the *Moon* and the *Nonagesime* at this time, by which we may find out *W* ☽ *e* the Complement of *W* ☽ ♀, the *Parallactical* Angle requir'd, to 90 *Degrees*; in the same way as we discover an Acute Angle of any other Oblique-angl'd Triangle, of which we have Two Sides and the included Angle. By which we may also find out *W* ☽ the Distance of the *Moon* from the *Vertex* at this time, if we have occasion for it. And if the *Parallactical* Angle at the *Moon* (or any other *Planet*) is requir'd to any other time, her

Longitude and Latitude being then known, and the Distance of the *Vertex* from the Pole of the *Ecliptic*, with the Longitude of the *Nonagesime* at that time, we may discover that Angle after the same manner.

PROBLEM V.

Having the Parallax *of Altitude of a* Star *or Planet, together with its* Parallactical *Angle; to find out its* Parallax *of Longitude and Latitude.*

SOLUTION.

Fig. 32. LET these be given in the *Moon* to the Time refer'd to in the preceding *Problem*: The *Moon* being depress'd by her *Parallax* from ☽ to *E* in the *Vertical* Circle *W* ☽ *E*, in the Triangle *E B* ☽ Right-angl'd at *B* the Side *E* ☽ is given, being her *Parallax* of Altitude, and the *Parallactical* Angle *E* ☽ *R*, with the Angle at *B* Right; from whence we may easily have ☽ *B* her *Parallax* of Longitude, and *E B* her *Parallax* of Latitude.

Q. E. I.

Scholium 1. If instead of the *Parallax* of Altitude, and *Parallactical* Angle, we had her *True* Longitude and Latitude, the Difference of Longitude 'twixt her and the *Nonagesimal* Point of the *Ecliptic*, and the Distance of that Point from the *Vertex*; we might thence find out her *Parallactical* Angle, and Distance from the *Vertex*, and having her Distance from the *Vertex* we might find out her *Parallax* of Altitude to that Distance, and from this and her *Parallactical* Angle her *Parallax* of Longitude and Latitude. For having the Distance of the *Nonagesime* from the *Vertex* *W N*

we

we have its Complement to 90 *Degrees*, the Distance of the *Vertex* from the Pole of the *Ecliptic* *We*, and having the *Moon's* Latitude we have its Complement to a *Quadrant* ☽ *e*; and having the Longitude of the *Moon* and *Nonagesime* we have their Difference *N e* ☽; from whence we may have *W* ☽ her Distance from the *Vertex*, and *W* ☽ *e* the Complement of her *Parallactical* Angle, to 90 *Degrees*; which last taken from 90 *Degrees* leaves us her *Parallactical* Augle; and having her Distance from the *Vertex*, and assuming her *Horizontal Parallax* from the *Astronomical Tables* fram'd for this purpose, we may easily have her *Parallax* of Altitude, that bearing the same *Ratio* to the *Horizontal Parallax* as the *Sine* of her given Distance from the *Vertex* does to the *Radius*: And having these, her *Parallax* of Longitude and Latitude may be found as in the *Problem*. And by the preceding *Problem* we know how to find out the Longitude of the *Nonagesime*, and its Distance from the *Vertex* to any Time assignable.

Schol. 2. The *Parallax* of the Altitude of the *Moon*, or of any other of the *Planets* which is near enough to the Centre of our *Earth* to have any *Parallax* at all, is nothing else but the Difference 'twixt the Places in the *Heavens* where it is view'd from the *Centre* and from the *Surface* of the *Earth*, measur'd by an Arch of a *Vertical* Circle intercepted 'twixt the Two Points it is view'd in from those Two Places: and this *Parallax* is greater the nearer the *Planet* to the Centre of our *Earth*; so that it is greatest of all in the *Moon*; and almost, if not intirely, insensible in the remotest *Planets*: and of the *Parallaxes* of Altitude in the *Moon* the *Horizontal* are the greatest, the rest decreasing as the *Sines* of their Distances from the *Vertex* decrease. The *Astronomers* have determin'd the *Moon's Horizontal Parallax* to all

the Parts of her *Orbit*, and from thence have deduc'd *Tables* of her *Parallaxes* to all the Altitudes which she can be found in at any time: So that having her *Horizontal Parallax* at any time we may thence have her *Parallax* of Altitude at the same time at any Distance which she shall be found in from the *Vertex*: And having this, and her *Parallactical* Angle, we may find out her *Parallax* of Longitude and Latitude, as in this *Problem*. For knowing the true Place in the *Heavens* where the *Moon* is view'd from the Centre of the *Earth* at any time, or her *True* Longitude and Latitude, from the *Tables* form'd by the *Astronomers* for this purpose, which gives the *Moon's* Places to an Eye at the Centre of the *Earth*; and the *Parallax* of her Altitude to the same Time from like *Tables*; as the *Parallax* of this Altitude shews us how much her *visible* is depress'd below her *True* Place, from this and the preceding *Problem*, we may discover how much this Depression alters her *True* Longitude and Latitude; that is, we may discover her *Visible* Longitude and Latitude at this time; and thence the Quantity, Duration, &c. of *Solar Eclipses*, the Occultations of *Fix'd Stars* by the *Moon*, &c. Thus if the *Moon's* true Place is at any time found by Calculation to be at ☽; her *True* Longitude being ♈ *e* ☽, and her True Latitude *f* ☽; Let her *Parallax* depress her to *E*, then will her *Visible* Place be at *E*; her *Visible* Longitude being ♈ *e E*, and her *Visible* Latitude *E K*; her *Parallax* at this time *adding* to her *True* Longitude the Arch *f K*, and *subtracting* from her *True* Latitude the Arch *B E*: For which investigating, *First*, all the Particulars requir'd in the preceding *Problem*, and thence getting the *Parallactical* Angle *W* ☽ ♀, or its Alternate *E* ☽ *B*, and her Distance from the *Vertex*, we may have *E* ☽ her *Parallax* of Altitude to that Distance from

from the *Tables*; and therefore, in the Triangle *E B* ☽, Right-angled at *B*, having *E* ☽ and *E* ☽ *B* and the Angle at *B* Right, we may easily have ☽ *B*, or *f e K*, which *added* to ♈ *e f* gives her *Visible* Longitude; and *E B* which *subtracted* from *K B* gives us her *Visible* Latitude. That is, her *Visible* Place: From whence we may have the Visible Quantity, Duration, *&c.* of any *Solar Eclipse* to any Latitude. The Reader will easily observe, that if the *Moon* has *True Southern* Latitude her *Apparent* or *Visible* Latitude will be more *Southern* than her *True*; or if her True *Northern* Latitude is less than the Arch of Difference of Latitude which her *Parallax* of Altitude gives her, her *Visible* Latitude will be *Southern*: If she is on the *Eastern* Side of the *Nonagesimal* Point of the *Ecliptic* her *Visible* will be greater than her *True* Longitude, if on the *Western* the contrary. And he will easily see from the Projection, that when she is upon the *Nonagesimal* Point of the *Ecliptic* her *Parallax* of Longitude will vanish, but that of Latitude will be the greatest, being then Equal to her *Parallax* of Altitude; but near the *Horizon* her *Parallax* of Latitude is always the least; and at, or near it, that of Longitude is the greatest.

Schol. 3. By this and the preceding *Problem* we are also taught how to correct any *Lunar Tables* by *Observation*: For Calculating by those *Tables* the *Moon's True* Longitude and Latitude to the Time when we are to make our *Observation*; and from thence deducing her *Visible* Longitude and Latitude; if we observe by the *Sextant* at that time her Distance from any Two *Fix'd Stars* whose Longitude and Latitude are known, making due Allowances for the Corruptions of their Distances by *Refractions*, according to *Schol.* 1. *Prob.* III. *Sect.* I. we may thence deduce her *Observ'd* Longitude and Latitude by the same *Scholium*:

lium: And if her *Visible* Longitude and Latitude, as given us by the *Tables*, are the same with those which we deduce from our *Observations*; then the *Tables* are good; if they differ then they want to be corrected. Or, noting her *Observ'd* Longitude and Latitude at any time, and Calculating her *Visible* Longitude and Latitude from the *Tables* at the same time, we may do the same thing. And repeating this Process from time to time quite round her *Orbit*, we shall find where the Fault lies in the *Tables* (if they are faulty) and withal how to correct it. In trying the *Tables* of the other *Planets* which have no *Parallax* with the Semidiameter of the *Earth*, if we *observe* their Places, as in the *Moon*, at any time; and Calculate from those *Tables* their *Heliocentric*, and from thence and the *Solar* and *Planetary Theories* deduce their *Geocentric* Places to that Time, we shall be enabled to do the same thing. But this I have only thought fit to hint at by the Bye, it being not directly to my present purpose.

PROBLEM VI.

Having the Latitude of the Place we are in, the Distance of the Sun *from the* North *Pole of the World at a given Time, and the Distance of the Parallel of* Twilight *from the* Vertex *at the same time; to find out the* Dawning *of the* Day *in the Morning, and the End of* Twilight *in the Evening*

SOLUTION *upon the Projection.*

IT is usually assum'd by the *Astronomers* (as they express themselves in the *Ptolemaic* Scheme) that the Light of the *Sun* reaches the *Horizon* in the Morning when the *Sun* itself is 16 *Degrees* below it, tho' some assume 18 *Degrees*, and some

19; and continues discernible in the Evening till it is depress'd so many *Degrees* below the *Horizon.* And whereas with the *Copernicans* the *Sun* does not move at all, but the *Earth*'s Diurnal Circumvolution upon its own *Axis* is the Foundation of the Diurnal *Phænomena*; hence it may be assum'd that the Day-light appears in the Morning when the *Vertex* of any Place wants at least 16 *Degrees* of emerging out of the *Obscure* into the *Illuminate* Part of the *Earth*'s *Disc*; and does not disappear in the Evening till it is immerg'd as many *Degrees* into the *Obscure* Part. If therefore we project a Parallel to the *Horizon* of the *Disc* (Perpendicular to the Plane of the Projection) at 16 *Degrees* Distance from it, or 74 *Degrees* from its nearest Pole *D*, according to the Directions of *Prop.* II. *Sect.* II. *Cap.* II. that Part of it which falls within the Plane of the Projection will be *A D E F*; *F* being the Point where our *Vertex* is when the Day begins to *dawn*, and *D* that where it is at the End of *Twilight*. And if we complete the Triangle *F p* ☉ having the Three Sides *F p*, *F* ☉ and *p* ☉, we may find the Angle *F p* ☉, the Time from *Noon* when the Day *dawns* by producing *p* ☉ and *F p* to Quadrants, and thro' the *Quadrantal* Points found, and an Opposite Point to either of them in their respective Circles completed striking a New Circle: For having the Centre of this New Circle we may have its *Inclination* to the Plane of the Projection, by *Coroll. Prop.* III. *Sect.* II. *Cap.* II. and thence its Pole within that Plane, by *Lemma* I. *Sect.* III. *Cap.* II. and thence we are enabled to discover the Value of that Arch of it which is intercepted 'twixt the two Quadrantal Points *First* found, by *Coroll.* 1. and 2. *Lemma* II. *Sect.* III. *Cap.* II. that is, to determine the Angle *F p* ☉, the Time before *Noon* when the Day dawns: Which is Equal Fig. 32.

to

to the Angle *D p* ⊙ the Time from *Noon* when the *Twilight* ends in the Evening.

Trigonometrical SOLUTION.

IN the Triangle *F p* ⊙, having *p* ⊙, 74° 27′ 29″: *p F*, 38° 28′ 00″; and ⊙ *F*, 106° 00′ 00″ we may have *F p* ⊙, by the *Eleventh* Case of Oblique-angled Spherical Triangles, after the following manner:

R : S. p F.	38°. 28′. 00″	9. 7938317
S. p ⊙.	74. 27. 29.	9. 9838222
F ⊙.	106. 00. 00.	9. 7776539
Z.	218. 55. 29.	0. 2223461
S. ½ *Z.*	109. 27. 44½.	9. 9744473
*X*ia.	3. 27. 44½.	8. 7809851
		18. 9777785
c.S. ½ *Fp* ⊙.	72°. 02′ 48″.	9. 4888892½
Fp ⊙.	144. 05. 36.	
15°)	144. 05. 36.	(9h. 36′. 22″.—
	135.	12. 00. 00.
	9. 05. 36.	2. 23. 38.

So that the End of the *Twilight* is at 9h 36′ 22″ in the Evening; and the *First* dawning of the Day is at 2h 23′ 38″ in the Morning.

Q. E. I.

PROBLEM VII.

Having the Latitude of the Place we are in, and the Right Ascension and Declination of any Point of the Ecliptic *or a* Fix'd Star, *given; to find out its Rising or Setting* Amplitude, *its Ascensional Difference, and consequently its Oblique Ascension.*

SOLUTION

SOLUTION *upon the Projection.*

IN the Scheme annex'd, which is a *Stereographic* Projection of the *Sphere* upon the Plane of the *Solstitial Colure* drawn according to the Directions of the *Second* Chapter; Let *VHEO* represent that *Colure* itself, *H ♈ O* the *Horizon* to the Latitude of *London*, *æ ♈ q* the *Equator*, *♋ ♈ ♑* the *Ecliptic*, *p ♈ π* the *Axis* of the World and the *Equinoctial Colure*, *V ♈ E* the *Prime Vertical*, and *S ♈ W* the Axis of the *Ecliptic*: And let it be requir'd to find out, *First* the Rising *Amplitude*, Ascensional Difference, and Oblique Ascension of the *First* Point of *Cancer*. The Diurnal Parallel of that Point is *♋ e f*, *e* the Point in the *Horizon* where it rises, *e ♈* or *e V ♈* its Rising *Amplitude* from the *East* towards the *North*; and *♈ x*, determin'd by its Circle of Right Ascension passing thro' the Points *e* and *x*, and the *Horizon*, its Ascensional Difference; which *subtracted* from its Right leaves us its Oblique Ascension. The Ascensional Difference being nothing else but the Difference 'twixt that Point of the *Equator* which culminates upon the *Meridian* with the *First* Point of *Cancer*, and that which rises with it above the *Horizon*: Which is here to be subtracted in order to find the oblique Ascension, because that Point of the *Equator* which rises with the first Point of *Cancer* comes to the *Horizon* before the Point of its Right Ascension, or that with which it culminates upon the *Meridian*. Fig. 35.

Now in the Triangle *♈ x e*, *First*, for *♈ e* the Rising *Amplitude* of the *First* Point of *Cancer*, if we lay a Ruler upon *V* and *e* it will cut the *Solstitial Colure* in *a*, making *E a* Equal to the Arch of the *Horizon* which *♈ e* represents: By *Prop.* II. *Sect.* III. *Cap.* II. wich will be found upon the

Sector,

Sector, open to the *Radius* of the Projection, to be within a few *Minutes* of 40 *Degrees*: And so much is the Rising Amplitude of the *First* Point of *Cancer* from the *East* towards the *North*. *Secondly*, for ♈ *x*, the Ascensional Difference, a Ruler laid upon *p* and *x* will cut the *Solstitial Colure* in β, making *x* β Equal to the Arch of the *Equator* which ♈ *x* represents: Which will be found (after the same manner) to be 33 *Degrees*, and 'twixt *Eight* and *Ten Minutes*: And so much is the Ascensional Difference of the *First* Point of *Cancer*; which *subtracted* from its Right leaves its Oblique Ascension.

Again, Let it be requir'd to investigate these Particulars with relation to the *First* Point of *Capricorn* the Diurnal Parallel of this Point of the *Ecliptic* is ♑ *d c*; *d* being the Point in the *Horizon* where it rises; *d* ♈, or *d V* ♈ its Rising *Amplitude* from the *East* towards the *South*; and ♈ *z* (determin'd as ♈ *x* before) its Ascensional Difference: Which may be had after the same manner as ♈ *e* and ♈ *x* were got in the former Case. And here the Ascensional Difference must be *added* to the *Right*, in order to find the *Oblique* Ascension, because that Point of the *Equator* which rises with the *First* Point of *Capricorn* comes to the *Horizon* after the Point of its Right Ascension, or that with which it culminates upon the *Meridian*.

Q. E. F.

Trigonometrical SOLUTION.

THese Particulars are altogether as easily, and with greater Accuracy, determin'd by *Trigonometrical Calculus*: For, for the Rising *Amplitude*, Ascensional Difference, and Oblique Ascension of the *First* Point of *Cancer*; in the Triangle ♈ *x e*, having *x e* the *Northern* Declination of that

that Point, 23° 29′ 00″; x ♈ e the Inclination of the Planes of the *Equator* and the *Horizon*, 38° 28′ 00″; and the Angle at x Right; we may easily find out ♈ e, and ♈ x, by the following Proportions:

S. e ♈ x : R : : S. e x : S. e ♈.
T. e ♈ x : T. e x : : R : S. ♈ x.

S. e ♈ x :	38° 28′. 00″.	9. 793831 —
R : : S. e x :	23. 29. 00.	19. 600409
S. e ♈.	39. 50. 08.	9. 806578

T. e ♈ x :	38°. 28′. 00″.	9. 900086 —
T. e x : :	23. 29. 00.	19. 637956
R : S. ♈ x.	33. 09. 05.	9. 737870

So that the Ascensional Difference of the *First* Point of *Cancer* is 33° 09′ 05″; which *taken* from 90 *Degrees* the Right Ascension of that Point, leaves its Oblique Ascension 56° 50′ 55″: And its Rising *Amplitude* from the *East* towards the *North*, is 39° 50′ 08″: Or, three Points, and 06° 20′ 08″ *Northerly*. From an Equal Triangle ♈ z d we find that the *First* Point of *Capricorn* has as much *Amplitude* from the *East* towards the *South*, as that of *Cancer* has from thence towards the *North*: And 33° 09′ 05″ added to the Right Ascension of the *First* Point of *Capricorn*, 270° 00′ 00″; gives us its Oblique Ascension, 303° 09′ 05″.

Q. E. I.

Scholium 1. After the same manner the *Astronomers* investigate the Ascensional differences of every Point of the first *Northern* Quadrant of the *Ecliptic*, which are determin'd by their *Northern* Declination; and *subtracting* 'em from their Right Ascensions

Aſcenſions frame Tables of their Oblique Aſcenſions: All the Triangles by which they inveſtigate 'em falling within the Triangle ♈ *x e*, and below the *Horizon*: And from them 'tis eaſy to make out the Oblique Aſcenſions of the *Second* Quadrant; every Point in it having the ſame Aſcenſional Difference with that Point in the *Firſt* Quadrant which has the ſame *Northern* Declination; and if we know their Aſcenſional Differences, and *ſubtract* 'em from their Right Aſcenſions, their Oblique Aſcenſions are alſo known. And in the *Southern* Quadrants they *add* the Aſcenſional Differences found in the *Firſt* Quadrants to the Right Aſcenſions of the Points in them which have the ſame Declination with any given Points in thoſe *Northern* Quadrants; in order to have their Oblique Aſcenſions. And ſo they get the Oblique Aſcenſions of all the Points in the *Ecliptic*.

Schol. 2. Every *Star* that riſes with any Point of the *Ecliptic* has the ſame Oblique Aſcenſion with that Point: Thus the *Star* at ∗, having *Northern* Declination *u* ∗, has for its Aſcenſional Difference *u* ♈, and conſequently riſes above the *Horizon* with the ſame Point of the *Equator*, which *e* the *Firſt* Point of *Cancer* riſes with: and having its Right Aſcenſion and Declination, we may find out its Aſcenſional Difference and Oblique Aſcenſion in the Triangle ♈ *u* ∗ after the ſame manner as we found out thoſe of the *Firſt* Point of *Cancer* in the Triangle ♈ *x e*: We may alſo after the ſame manner have the *Azimuth* which it riſes upon. If we would therefore know what Point of the *Ecliptic* any *Star* riſes with, having its Right Aſcenſion and Declination we may have its Aſcenſional Difference and Oblique Aſcenſion; and finding out to what Point of the *Ecliptic* that Oblique Aſcenſion belongs, from the Tables fram'd by

by the *Astronomers* for this purpose, that is the Point which rises with the *Star* propos'd.

Schol. 3. The Intelligent Reader will easily see that when the Ascensional Difference is *subtracted* from the Right in order to have the Oblique Ascension, if it is *added* to the Right it will give us the Oblique Descension: Because that Point of the *Equator* which *sets* (for Instance) with the *First* Point of *Cancer* comes to the *Horizon* before the Point of its Right Ascension, or that with which it culminates upon the *Meridian*. And when it is *added* to the Right for the Oblique Ascension, if it is *subtracted* from the Right it will leave us the Oblique Descension: Because that Point of the *Equator* which *sets* (for Instance) with the *First* Point of *Capricorn*, comes to the *Horizon* after the Point of its Right Ascension, or that with which it culminates upon the *Meridian*.

Schol. 4. If we have a mind to know the Length of the Day in our Latitude when the *Sun* is in the *First* Point of *Cancer*, his Ascensional Difference in that Point turn'd into Time, and *subtracted* from *Six* in the Morning, gives us the Time of his *Rising*; and *added* to *Six* in the Evening the Time of his *Setting*; which being got we have the Length of the Day. Thus 33° 09′ 05″ the Ascensional Difference of the *Sun* in the *First* Point of *Cancer*, turn'd into Time is 2h 12′ 36″, which *subtracted* from *Six* leaves 3h 47′ 24″, the Time when the *Sun rises* at *London*, in this Point of the *Ecliptic*; and *added* to *Six* it gives 8h 12′ 36″, the Time when the *Sun sets* in the same Latitude. And if we *add* the same to *Six* in the Morning when the *Sun* is in the *First* Point of *Capricorn*, we shall find that he rises then at 8h 12′ 36″; and if we *subtract* it from *Six* in the Evening, he *sets* at 3h 47′ 24″ in that Point. So that the longest Day at *London* is 16h 25′ 12″; and the shortest 7h 34′ 48″. And after

after the same manner. may we find out these Particulars to every intermediate Point in the *Ecliptic*.

Schol. 5. Besides the Determination of the Time when any *Star* rises above the *Horizon* on any Day in the Year, which is got from its Right Ascension and Declination known, and the Difference of Right Ascension 'twixt it and the *Sun* at that Time; by *Schol. ad Prob.* I. *Sect.* II. By which *Scholium* also we may with Equal Ease determine when it sets: Which are properly call'd the *Astronomical* Rising and Setting of a *Star*. We are requir'd sometimes (in order to understand some Passages in the antient Poets, Natural Historians, Writers *de re Rustica*, &c.) to determine at what Time a *Star* rises or sets when the *Sun* rises, which is call'd its *Cosmical* Rising and Setting. At other times we are call'd upon to determine at what Time a Star rises or sets with the setting *Sun*: which is call'd its *Achronical* Rising or Setting. And, *Lastly*, sometimes it is demanded of us when a *Star* which lately, by reason of its Nearness to the *Sun*, was invisible, is so far left by the *Sun* in his Annual Motion, as to emerge out of his Rays, and become visible; which is call'd its *Heliacal* Rising; or, when that, or any other *Star*, which has lately been Visible is so far overtaken by the *Sun* in his Annual Motion as to be absorp'd in his Rays, and become Invisible which is call'd its *Heliacal* Setting. And (if we are Masters of the *Solar Theory*) the Two former of these, *viz.* the *Cosmical* and *Achronical* Rising and Setting of a *Star*, may be determin'd by its Oblique Ascension and Descension known, and the Points of the *Ecliptic* with which it rises and sets, which are got from its Oblique Ascension and Descension. For let a *Star* whose *Cosmical* Rising is requir'd have the same Oblique

Ascension with the *First* Point of *Cancer*, or any other Point of the *Ecliptic*; then it rises with that Point: and consequently, when the *Sun* is in the *First* Point of *Cancer*, or that Point of the *Ecliptic* in which he has the same Oblique Ascension with the *Star*, that *Star* rises *Cosmically*: Let a *Star* have the same Oblique Descension with any Point of the *Ecliptic*, then when the *Sun* rises in the Opposite Point, that *Star* sets Cosmically. If a *Star* has the same Oblique Descension with the *First* Point of *Cancer*, or any other Point in the *Ecliptic*, it sets with that Point; and consequently when the *Sun* is in the *First* Point of *Cancer*, or that Point of the *Ecliptic* in which he has the same Oblique Descension with the *Star*, that *Star* sets *Achronycally*: If a *Star* has the same Oblique Ascension with any Point of the *Ecliptic*, then, when the *Sun* is in the Opposite Point, that *Star* rises *Achronycally*. Whence we may determine the Times of the *Cosmical* and *Achronycal* Risings and Settings of all the *Stars* in the Heavens, whose Right Ascensions and Declinations, and consequently Oblique Ascensions, are known, if we know the *Solar Theory* at the same time. The *Heliacal* Risings and Settings of the *Stars* are investigated in the following *Problem*.

PROBLEM VIII.

Having the Latitude of the Place we are in, the Points of the Ecliptic *with which a* Star *Rises and Sets, and the Altitude of the* Nonagesime *when those Points are upon the* Horizon; *to find out the Points of the* Ecliptic *the Sun must be in to make the* Star *Rising or Setting appear just free from the Solar Rays; and from thence the Times of its* Heliacal *Rising and Setting.*

Solution.

THE excellent *Kepler's* Answer to this Question, "How far ought the *Sun* to have "departed from a *Fix'd Star* or *Planet*, to permit us to see it free from his Rays?" is this: "That tho' the Solution of this Question admits of "great variety; differing according to the Brightness of the fix'd *Stars* or *Planets*; according to "the State of the Air, which is different in different Climates, and in the same in different Seasons; according to the different Latitudes which "the Question is propos'd in, in which the Twilight "is of different Power and Duration; yet the "*Astronomers* have assum'd some Particulars, in "order to the Solution of this *Problem*, which "have been found very nearly consonant to *Observations*: viz. *First*, That the *Twilight* commences in the Morning, and concludes in the "Evening, when the *Sun* is depress'd in a *Vertical* "Circle 19 *Degrees* below the *Horizon*; tho' here "*Tycho Brahe* requires but 16 *Degrees* Depression "at the *Equinoxes*; others 18 *Degrees*. *Secondly*, That we cannot see the smallest *Fix'd Stars* in "Parts of the *Horizon* where the *Sun* rises or "sets, but when the *Sun* is depress'd 18 *Degrees* "below the *Horizon*. For *Stars* of the *Sixth* "Magnitude, they depress the *Sun* 17 *Degrees*; "For those of the *Fifth*, 16; For those of the "*Fourth*, 15; For those of the *Third*, 14; For "those of the *Second*, 13; And for those of the "*First*, 12: And as to the *Planets*, they assert, "That we cannot see *Mars*, till the *Sun* is "$11^{\circ}\ 30'\ 00''$ below the *Horizon*; *Saturn*, till "he is 11 *Degrees*; *Jupiter* or *Mercury*, till 10; "and *Venus*, till he is depress'd 5 *Degrees* below "the *Horizon*. Tho' the different Distances of

"the

"the *Planets* from the Centre of the *Earth*, in "different Points of their *Orbits*, seem to require "a greater variety in this Matter than is here "specify'd." *Vid. Kepleri Epit. Astronom. Copernican. lib. 3. pag. 369, 370. Edit. Lent. ad Danubium.*

Let the Solution therefore of this *Problem* relate to a *Star* of the *First* Magnitude; which is seen when the *Sun* is depress'd 12 *Degrees* below the *Horizon*: Having the *Solar Theory*; and the Points of the *Ecliptic* with which this *Star* rises and sets; we know at what Time of the Year the *Sun* is in those Points, and consequently when this *Star* rises and sets with the *Sun*: And knowing when this *Star* rises and sets with the *Sun*, and the Rising and Setting Points of the *Ecliptic* at those Times, we have its *Nonagesimal* Degree from the *Horizon* in both Cases; and may have the Altitude of the *Nonagesime* at the same time, either by *Prob.* IV. *Sect.* II. or from Tables fram'd by the *Astronomers* for this Purpose; and having these *two*, *viz.* those Points of the *Ecliptic* in which this *Star* rises or sets with the *Sun*; and the Altitude of the *Nonagesime*, or the Inclination of the Planes of the *Ecliptic* and *Horizon* to these Points Rising or Setting: In the Scheme annex'd, Let *H b r O* represent the *Horizon*, *e b S c* the *Ecliptic*, *V r S n* a *Vertical* Circle, in which *S* the *Sun* is depress'd 12 *Degrees* below the *Horizon*; *b* being the Point of the *Ecliptic* where the *Sun* is when the *Star* rises or sets with the *Sun*: Therefore in the Triangle *b r S* Right-angl'd at *r*, we have *r S* 12 *Degrees*, the requisite Depression of the *Sun* below the *Horizon* to free the *Star* from his Rays, or the Point the *Sun* must be in to a *Star* which rises or sets with the Point of the *Ecliptic* at *b* to make it first *Heliacally* visible when it rises, *Fig.* 36.

or

or from which it disappears when it sets *Heliacally*; *sbr* the Altitude of the *Nonagesime*, or the Inclination of the Planes of the *Ecliptic* and *Horizon*: and the Angle at *r* Right, as form'd by the Intersection of a *Vertical* Circle and the *Horizon*: And consequently may easily find *bs* the Arch of the *Ecliptic*, which the *Sun* must run from the Time that this *Star* rises with him to its *Heliacal* Rising, if *b* is the Point of the *Ecliptic* with which it rises; or the same *bs* the Arch of the *Ecliptic* which the *Sun* must have run from the Time when the Star set *Heliacally* to its Setting with the *Sun*, if *b* is the Point of the *Ecliptic* with which the *Star* sets. Which in the former Case, *added* to the Point of the *Ecliptic* in which the *Sun* is when the *Star* rises with him, gives the Point he is in at its *Heliacal* Rising: And in the latter Case, *subtracted* from that Point of the *Ecliptic* the *Sun* is in when the *Star* sets with him, leaves the Point he is in at the same *Star's Heliacal* Setting. And knowing the Times when the *Sun* is in those Points of the *Ecliptic* from the *Solar Theory*, we have those of the *Star's Heliacal* Rising and Setting.

Q. E. I.

Scholium. The antient *Greeks* and *Romans* made use of a mix'd *Lunæ-Solar* Year, which was not so well adjusted to the *Tropical* Year, but that sometimes in their Reckonings they were before, and sometimes after the *Tropical* Time: And whereas the Returns of the Seasons do not depend upon any erroneous *Calendar*, but upon the Approaches of the *Sun* to the *Tropics* or the *Equinoctial* Points; therefore to determine the Approaches of the Seasons proper for the Affairs (whether Rustic, Religious, Civil, or Military) to be transacted in them; they made use of some of

of these Rsiings or Settings of the principal *Fix'd Stars*, as *Criteria*, by which to judge of the Commencements of the Seasons they were to treat of. This every one knows, who is in any measure acquainted with the antient Poets, Natural Historians, Writers *de Re Rustica*, &c. And if we wou'd know whether any antient Author is accurate in his determining the Risings or Settings of any *Fix'd Stars*, and what sorts of Risings and Settings he refers to: Having the Longitude and Latitude of an assign'd *Star*, to the Time refered to by that antient Author, (which may be had from its present Longitude and Latitude, their Latitude continuing always invariably the same, and their Longitude being now 50 *Seconds* forwarder for every Year of the Time elaps'd 'twixt that the Author refers to and the present) together with an *Ephemeris* of the *Sun*'s Motions, calculated to the Times the Writer we refer to liv'd in, in the *Julian* Year extended backward as far as requisite: If the Writer gives us the Day of the *Roman* Year, and the Rising or Setting of the *Star* in an assign'd Latitude, we must seek in the antient *Ephemeris* the *Sun*'s Place to that Time, and the Point of the *Ecliptic* which rises or sets with that *Star*; and comparing the *Sun*'s Place at that Time with that Point of the *Ecliptic*, according to the Direction of the two preceding *Problems*, we shall find what kinds of Risings or Settings this Author must mean, and whether he is accurate in his Determinations. Hence also we may learn the different Times of these Risings and Settings of the *Stars* at this Day, from those assign'd 'em by antient Authors, *&c.* But these Things are in some measure by the bye being only hinted to shew the principal Use of these two last *Problems*, as they are subservient to the full Under anding of innu-

merable Passages of the Antients relating to the Risings and Settings of the principal *Fix'd Stars*; by which they determin'd the Commencements of the Times set apart for these or those Religious or Political Uses, and of the Seasons proper for this or that part of Agriculture, *&c.* He who wou'd see this Subject treated at large, may consult the latter end of the *Third* Book of *Kepler's Epitome of the Copernican Astronomy*, from *pag.* 372 to 398 inclusive.

PROBLEM IX.

To solve any of the preceding Problems, *which we have solv'd on the* Stereographic *Projection of the* Sphere *upon the Plane of the* Ecliptic, *upon a like Projection on the Plane of the* Solstitial Colure.

SOLUTION.

First, LET the *First Problem* of the First Section of this Chapter, which I have already solv'd upon the Plane of the *Ecliptic*, be requir'd to be solv'd upon the Plane of the *Solstitial Colure*:
Fig. 35. That *Problem* requires, "That having the *Sun's* " Place in the *Ecliptic*, and the Inclination of the " Planes of the *Equator* and the *Ecliptic* to each " other; we find out his Right Ascension from " the *First* Point of *Aries*, his Distance from the " *North* Pole of the World, and the Angle which " the *Meridian* passing through the *Sun* at that " Place makes with the *Ecliptic*:" Which may be discover'd in the Projection upon the Plane of the *Solstitial Colure*, after the following manner.

Let the *Sun* be suppos'd in the *First* Point of *Gemini*, therefore in the Figure it will be at ♊ in the Representative of the *Ecliptic* ♋ ♈ ♑: Through which and the two Poles of the *Equator* *p* ♓ a

great Circle being struck, it will cut off ♈ φ the Right Ascension of the *Sun* in the *First* Point of *Gemini*, φ ♊ being its Declination *North*, ♊ *p* its Distance from the *Pole*; and φ ♊ ♈ the Angle which the *Meridian* passing through the *Sun* in this Place makes with the *Ecliptic*. And *First*, for ♈ φ; a Ruler laid upon *p* and φ will cut the *Solstitial* Colure at the Point γ, making π γ equal to the Arch of the *Equator* which ♈ φ represents; and therein giving us the Right Ascension of the *Sun* in the *First* Point of *Gemini*; by *Prop*. I. *Sect*. III. *Chap*. II. *Secondly*, for φ ♊ or ♊ *p*; the Centre of the great Circle *p* ı *r* being found at χ, its Inclination to the Plane of the Projection may be found; by *Coroll. ad Prop*. III. *Sect*. II. *Cap*. II. and its *Pole* which falls within that Plane by *Lemma* I. *Sect*. III. *Cap*. II. Which being had, we are enabled to discover the Value of φ ♊ the Declination of the Sun in this Point of the *Ecliptic*; by *Coroll*. II. *Lemma* II. *Sect* III. *Cap*. II. the Complement of which to 90 *Degrees* is his Distance in this point of the *Ecliptic* from the *North* Pole of the World. And *Thirdly*, for *p* ♊ ♋, or its Alternate ♈ ♊ φ, the Angle which the proper *Meridian* to this Place of the *Sun* makes with the *Ecliptic*; if we take the Representative of a Quadrant of the Circle *p* ♊ π from ♊ to δ, and another in the *Ecliptic* from ♊ towards ♑; and through these two Points and the opposite Point to that found in the *Ecliptic* strike a great Circle; having its Centre, we have its Inclination to the Plane of the Projection, and its Pole within that Plane, and by that we are enabled to find the Value of the Arch intercepted 'twixt the Point at δ and the *First* Point found in the *Ecliptic*; or of the Angle φ ♊ ♈, which the proper *Meridian* passing thro' the *Sun* in the *First* Point of *Gemini* makes withhe *Ecliptic*.

Secondly, Let the *Third Problem* of the *First* Section of this Chapter be requir'd to be solv'd upon the Plane of the *Solstitial Colure*: That *Problem* requires, "That having the Right Ascen-"sion and Declination of two *Fix'd Stars*, and "the Distance of a third, whether *Fix'd Star* or "*Planet*, from each of them; we find out the "Right Ascension and Declination of that *Fix'd* "*Star* or *Planet*:" Which *Problem* may be solv'd upon the Plane of the *Solstitial Colure* after the following manner.

Having the Right Ascension and Declination of a *Star*, 'tis easy to lay it down upon the Plane of the Projection: For Instance, the *Star* at ♃; for knowing its Right Ascension, we know how many Degrees it is distant in the *Equator* from the *Last* Point in ♓; which laid off from the Centre of the Projection in the Representative of the *Equator* to ϰ, gives us the Points *p*, ϰ and π, through which to strike the Circle of Right Ascension proper to this *Star*; and having its Circle of Right Ascension, we have the Distance of the Centre of that Circle from that of the Projection, its Inclination to the Plane of the Projection, and its Pole which falls within that Plane; from which and the *Star*'s Declination known, we may discover the Arch of the Circle of Right Ascension representing that Declination; and consequently may project the *Star* at ♃ itself. And after the same manner may we project the *Star* at ☿. Or if we project the Parallel of the Declination of either of these *Stars*, it will cut their Circle of Right Ascension in the Point where each *Star* is to be Projected.

And having the two *Fix'd Stars* from which (for instance) the *Moon* is *Observ'd*, laid down upon the Plane of the Projection; if we strike a great Circle through the two *Stars* at ♃ and ☿,

and

and the Point of the Circle of the Right Ascension of (for Instance) the *Star* at ☿ opposite to that where the *Star* is, *viz.* *A*; it will be *A* ♃ ☿ *B*, having the Point at *V* for its Centre: In which it is easy to discover, by the Rules already laid down and so often refer'd to, its Inclination to the Plane of the Projection, and its Pole within that Plane; and and consequently the Value of the Arch ♃ ☿, or the Distance of the two *Stars* from each other.

Let this New Circle be taken for a *Primitive*, and let the *Moon* ♃e *observ'd* 40 *Degrees* distant from the *Star* at , and 30 *Degrees* from that at ☿. If we take the Point at ♃ in this New *Primitive* Circle for the Pole of a Lesser Circle perpendicular to its Plane, 40 *Degrees* distant from that Pole; and project that Lesser Circle according to the Directions of *Prop.* II. *Sect.* II. *Cap.* II. laying off the *Secant* of 40 *Degrees*, to the *Radius* of the New *Primitive*, from *V* to *D*; and making the *Radius* of the Lesser Circle equal to the Tangent of the same Arch; we shall find *FHG* as much of this Lesser Circle as falls within the New Plane of the Projection: In some Point of which the *Moon's* Place in the Projection will be found; as she is 40 *Degrees* of a Great Circle of the *Sphere* distant from the *Star* at ♃, and consequently in a Parallel 40 *Degrees* distant from that *Star* assum'd as the Pole of a Great Circle; and as *FHG* represents so much of such a Parallel as falls within the Plane of the Projection.

And if we take the *Star* at ☿ for a New Pole of a Lesser Circle perpendicular likewise to the Plane of the Projection, or Parallel to a Great Circle which is perpendicular to it, the Lesser Circle being 30 *Degrees* distant from that Pole; and project that lesser Circle after the same man-

ner

ner that we did the former; we shall find so much of it as falls within the Plane of the Projection to be represented by *K* ☽ *M*: In some Point of which also the *Moon*'s Place in the Projection will be found; as she is 30 *Degrees* of a Great Circle of the *Sphere* distant from the *Star* at ☿; and as all Parallel Circles cut off Arches of the Great Circles which pass through their Poles equal to their own Distances from those Poles, and consequently their Representatives must do so by the Representatives of those Great Circles. And if the *Moon* is to be found in the Parallel *F H G*, and in the Parallel *K* ☽ *M* at the same time, she must be in the Point of their Intersection, *viz.* at the Point ☽.

And having the *Moon*'s Place in the Projection, if we strike a Circle of Right Ascension through the two Poles of the World *p* and *π*, and the Point at ☽, where she is found; we may easily find her Right Ascension and Declination to the Time when she was observ'd at the fore-mention'd Distances from the two *Stars* given. As every one must see without any necessity of my specifying the Process in this Place.

The Reader who understands how this *Problem* is projected in the Plane before us, cannot be at a loss to discover how it might have been projected upon the Plane of the *Ecliptic*: I chose not to project it there, because it would have render'd the *Scheme* too confused and intricate: Neither do I complete the Triangle made by the *Moon* and the two *Stars* here; because after her Place in the Projection is found, her Right Ascension and Declination, or her Longitude and Latitude, may be easily determin'd without it. Those who have a mind to do it, may easily strike a Great Circle through the *Moon*, the *Star* at ♃, and the Point in the Great Circle *A* ♃ *B* opposite to that in which

which the *Star* at ♃ is plac'd; and another through the *Moon*, the *Star* at ♀, and the Point in the Circle *A* ☿ *B* opposite to that in which the *Star* at ☿ is found; and the thing is done.

Thirdly, Let the *Third Problem* of the *Second* Section of this Chapter, which I have already solv'd upon the Plane of the *Ecliptic*, be requir'd to be solv'd upon the Plane of the *Solstitial Colure*. That *Problem* requires, "That having the *Sun*'s Place in "the *Ecliptic*, and his Declination from the *Equator* "in that Place; together with the Latitude of "the Place we are in, and his Distance from the "*Vertex* by *Observation*; we find out the Time of "the Day when that *Observation* was made, and "the *Azimuth* the *Sun* was then upon." Which *Problem* may be solv'd upon the Plane of the *Solstitial Colure* after the following manner.

Let the *Sun* be in the *First* Point of *Cancer*, and *observ'd* 40 *Degrees* above the *Horizon*, or 50 *Degrees* distant from the *Vertex*: As he is in the *First* Point of *Cancer*, his Semi-diurnal Parallel is ♋ II *s*; and as he is 50 *Degrees* from the *Vertex*, he is in a Parallel of Altitude which is so far distant from the *Vertex* as its Pole; which being Perpendicular to the Plane of the Projection, is projected (according to the Directions of *Prop.* II. *Sect.* II. *Cap.* II.) into *a H x*; and if he is in both these Parallels, he must be in their Point of Intersection, *viz.* at ☉. And having thus got the Place which the *Sun* is in, if we strike an *Azimuth* Circle through that Point and the two Poles of the *Horizon V* and *E*, it will cut off an Arch of the *Horizon*, which measur'd, will give us his Amplitude from the *East* towards the *South*, if the *Observation* is made in the Morning; and from the *West* towards the same, if it is made in the Afternoon. And another great Circle struck through

through the Poles of the World and the same Point, (*viz.* $p \odot \pi$) will cut off $z\, \alpha$ in the Representative of the *Equator*; which measured, and turn'd into Time, will give us the Time before or after Noon, (according to the Time when we made our *Observation*) when the *Sun* is 50 *Degrees* from the *Vertex*, or 40 *Degrees* above the *Horizon*, in the *First* Point of *Cancer*.

Q. E. F.

Scholium 1. After the same manner may we discover the Time of the Day, and the *Azimuth* the *Sun* is upon at that Time, from his *Observ'd* Distance from the *Vertex*, and the Latitude of the Place we are in, in whatever Point of the *Ecliptic* we meet with him: For knowing the Declination of that Point, and projecting the Parallel of that Declination; and knowing the *Sun's* Height, and Projecting its Parallel; these intersect each other in his Place in the Projection; through which and the proper Poles striking a *Vertical* Circle, and a Circle of Right Ascension, they give us the Arches in the *Horizon* and the *Equator*, which enable us to determine these Particulars. And in like manner may we solve any other *Astronomical* Problems upon this Projection: Thus if we would know at what Time the *Sun* rises with us in the *First* Point of *Cancer*, and the *Azimuth* which he rises upon; as he rises then at ε, the Arch $\Upsilon\, \varepsilon$ determines the Point of the Compass from the *East* towards the *North* which he rises upon; and the Angle $\Upsilon\, p\, \varepsilon$, or the Arch $\Upsilon\, \varkappa$ turn'd into Time, lets us know how much before *Six* he rises in the Morning; both which may easily be had: and after the same manner may we know when and upon what Point he sets. If we wou'd know upon what *Azimuth*, and at what Distance from the *Vertex* the *Sun* is (when in the *First* Point

Point of *Cancer*) at *Six* in the Morning; as he is then at ♀ in the *Six-a-Clock* Hour-Circle, a Circle ſtruck through that Point and the two Poles of the *Horizon* will determine the *Azimuth* he is then upon; and finding the Pole of that Circle which falls within the Plane of the Projection, we may eaſily diſcover the Value of that Arch of it which is intercepted 'twixt the *Sun* at ♀ and the *Horizon*, the Complement of which to 90 *Degrees* is his Diſtance from the *Vertex* at that Time: And the ſame Proceſs will tell us upon what *Azimuth* and at what Diſtance from the *Vertex* he is on the ſame Day at *Six* in the Evening. And in like manner may the Reader project and ſolve any other *Aſtronomical* Problem whatever, reſulting from the Diurnal Motion of the *Earth* upon its own *Axis*, &c. upon this Plane: Or upon the Plane of any other Great Circle whatever which the *Sphere* ſhall be either *Orthographically* or *Stereographically* projected upon. I have given no Inſtances of the Solutions of any *Problems* upon *Orthographic* Projections, becauſe of the Difficulty of Drawing the *Ellipſes* with ſufficient Accuracy by which they muſt be ſolv'd: But the Intelligent Reader muſt be able to do it for himſelf by this time; and to ſolve any *Problem* of the *Sphere* upon an *Orthographic* Projection upon the Plane of the *Solſtitial Colure*, as well as upon a *Stereographic* Projection of it upon the ſame Plane. And he who can do all theſe things, *viz.* Solve any *Aſtronomical* Problems aſſignable, upon any either *Orthographic* or *Stereographic* Projection of the *Sphere* upon the Planes of any of its great Circles, muſt be own'd to be ſufficiently acquainted with the Uſe of the Projection of the *Sphere in Plano*, in the Solution of *Aſtronomical* Problems.

Schol.

Schol. 2. From what has been ſaid, it were eaſie to deduce the Projection of all the reſt of the Caſes of both *Right* and *Oblique-angled Spherical* Triangles, as well as thoſe which we have had occaſion for: As every one muſt ſee, who is throughly acquainted with the Contents of the preceding Chapter, and the Application of them in this. I have not room to do it here; but muſt leave it as an Exerciſe to the Intelligent Reader, who has either occaſion for, or takes delight in theſe Matters.

CHAP.

CHAP. IV.

The Use of the Projection of the SPHERE in Plano, *in the Delineation of* MAPS *of the* WORLD, *or of* particular Countries; *and in the Solution of* Geographical *Problems.*

AS GEOGRAPHY, in its utmost Latitude, has for its Object no less than the Figure and Magnitude of the *Earth* we live upon; its Motion, and Position with respect to the other Bodies we meet with in the great System of the *World*; the Matter it is made of, and the Extent, Position, &c. of the Solids and Fluids which compose its Surface; the different Lengths of the Days and Nights, Variations of the Seasons, &c. in several Parts of it: In which Case it takes in a great part of *Spherical Astronomy*, *Natural History*, *Meteorology*, *Natural Philosophy*, and the like. And as in a more restrain'd and stricter Sense, the *Lands* and *Seas*, *Islands* and *Continents*, *Cities* and *Countries*, &c. which we must meet with upon the Surface of our *Earth*, with their respective Extents, and the Situation of all or any of these with respect to each other, or to any Point or Line either upon the *Earth* or in the *Heavens*, with which they may all be compar'd, are its Principal Object: In which Case also, as far as any Places upon the *Earth* are compar'd, with respect to their Situation, and the *Phænomena* thence

thence resulting, with the *Sun*, or any other Points in the *Heavens*, the *Geographer* is forc'd to have recourse to *Spherical Astronomy* for the Solution of a great many *Problems* relating to the Lengths of the Days and Nights, Risings and Settings of the *Stars*, &c. which are proper to those Places. So, in the strictest sense of all, the *Lands* and *Seas*, *Islands* and *Continents*, *Cities* and *Countries*, *Isthmus's*, *Mountains*, *Peninsula's*, &c. to be met with upon the *Earth*'s Surface, with their respective Extents, and the Situation of all or any of them with respect to each other, or to any Point or Line upon the Surface of the *Earth* with which they may all be compar'd, are the principal Object of *Geography properly so call'd.* And in this sense I shall take the Object of *Geography* in this Place: Having already, upon the Projection of the *Sphere in Plano*, solv'd the principal *Problems* relating to any Places upon the Surface of our *Earth*, compar'd as to Situation with the *Sun*, or any other of the *Fix'd Stars* in the *Heavens*. And, in order to shew the Use of the Projection of the *Sphere in Plano* in the Delineation of *Maps* of the *World*, and of *particular Countries*, and in the Solution of *Geographical* Problems, I must assume the following *Postulatum*.

POSTULATUM:

The Earth *is of a* Spherical *Figure: Or, the whole Compages of the* Terraqueous *System, of which our* Earth *is compos'd, is form'd into one large* Sphere.

Scholium. The Sentiments of some of the antient Philosophers, with relation to the Figure of the *Earth*, nay, and of some of the old Fathers too

too, were very extravagant and absurd: But all the Mathematicians from the earliest Antiquity down to these present Times have unanimously agreed, that it is of a *Spherical* Figure. The Arguments for this Hypothesis may be seen in a great many Authors, and therefore need not be repeated in this Place: I shall only observe here, That by the late great Discoveries in Natural Philosophy, it appears, that the *Earth* is not a perfect *Sphere*, but an *Oblate Spheroid*, or such a Solid as is generated by the Circumrotation of an *Ellipsis* upon its shorter *Axis*; having its *Equatorial* Diameter something longer than the *Axis* of its Diurnal Circumvolution. However, in *Geography* we may consider it as a perfect *Sphere*, that little which it deviates from it being of no Consideration in *Geographical* Calculations, and look upon it as truly represented by what we call the *Terrestrial Globe*, with all the *Lands* and *Seas* describ'd upon them. And thus considering it, it will be easy to shew the Use of the Projection of the *Sphere in Plano* in the Delineation of *Maps* of the *World*, and of *Particular Countries*; and in the Solution of *Geographical* Problems.

SECT. I.

The Solution of such Problems *as relate to the Delineation of* Maps *of the* World, *or of particular* Countries, *upon a Projection of the* Sphere in Plano. Sect. I.

PROBLEM I.

Having the Longitude and Latitude of any Place upon the Surface of the Earth; *To find out its*

Place in the Stereographic *Projection of the* Sphere *upon the Plane of any one of its* great *Circles.*

SOLUTION.

LET the *Sphere* be projected upon the Plane of the *Meridian* of the Place we live in, (for Instance, upon that of the *Meridian* of *London*;) and let the Projection, *Fig.* 30. represent the Plane of our *Meridian* with the *Sphere* projected upon it, according to the Directions of the *Third* Example, *Sect.* IV. *Cap.* II. then will the Place of *London*, in this Projection, be found in the *great* Circle ♈ *p* ♎ ♑: And *p* representing the *North* Pole of the *Globe*, and *London* being 38° 28′ 00″ from that Pole; the *Chord* of 38° 28′ 00″ to the *Radius* of the Projection, laid off from *p* to *L*, gives *L* the *Place* of *London*. Let it be requir'd to find out the *Place* in the Projection of *Vienna* in *Austria*, whose Latitude (according to Monf. *de la Hire*'s Catalogue) is 48° 22′ 00″ *North*, and the Difference of *Longitude* 'twixt it and *London* is 17° 07′ 30″, *Vienna* being so much *Easterly* of *London*: The Circle of Longitude therefore of *Vienna* being *Inclin'd* to the Plane of the Projection 17° 07′ 30″, the Tangent of 17° 07′ 30″ set off from *a* to *q*, gives *q* the Centre of the Circle of Longitude of *Vienna*; by *Prop.* III. *Sect.* II. *Cap.* II. And as that *great* Circle passes through both the Poles of the *Globe*, and consequently its Representative through both theirs, the Circle *p r* ♑ is the Circle of Longitude of *Vienna*, in some Point of which consequently that Place is to be found: In order to which, I have assum'd, that the Latitude of *Vienna* is 48° 22′ 00″ *North*, or that it is so far distant from the *Equator* represented by ♈ *a* ♎, in its Circle of Longitude *p r* ♑, 'twixt *r* and *p*. The Pole of this Circle of Longitude

gitude which falls within the Plane of the Projection, is distant from the Centre of the Projection 08° 33′ 45″; by *Lemma* I. *Sect.* III. *Cap.* II. the Tangent of which, to the *Radius* of the Projection, set off from *æ* to *g*, gives *g* that Pole; from which a Line drawn to the Point *x*, which is 48° 22′ 00″ distant from the Point ♎ in the *Equator*, will cut the *Meridian* of *Vienna* in the Point *v*; which is the Place of that City in this Projection: *v r* being the Representative of 48° 22′ 00″ of its *Meridian*; by *Coroll.* 1. *ad Lemma* II. *Sect.* III. *Cap.* II.

Q. E. F.

Scholium 1. In like manner must the Reader, who throughly understands the *Second* Chapter of this Book, be able to solve this *Problem* upon the Projection of the *Sphere* upon the Plane of any other of its *great* Circles. I have instanc'd in the Plane before us, because 'tis the Plane of some *Meridian* or other that most of the *Maps* of the *World* are drawn upon; as the Positions of all the Countries upon the Surface of the *Globe*, with respect to each other, are best represented upon these Planes. And from this single *Problem* the Reader must be enabled to draw a *Map* of the *World* upon the Plane of what *Meridian* he is willing to make choice of; for Instance, that of *London*: For as we have found the *Place* of *Vienna*, after the same manner may we find the *Places* of any other Cities, Towns, Forts, Promontories, &c. whose Longitudes and Latitudes are known, in either *Hemisphere*, which is made by the Plane of the *Meridian* of *London*; as every one must see, without any further Illustration of the Matter. The Reader must also hence be acquainted, what are the grand Requisites by which we are enabled to draw true *Maps* of the *World*,

 or

or to represent the whole Surface of the *Earth* upon a Plane, as it ſhall appear upon the Plane to the *Eye* in one of the Poles of whatever *Firſt Meridian* we pitch upon, *viz.* an accurate Knowledge of the Longitude and Latitude of all the Remarkable Places upon the Surface of the *Earth.* How to get the Latitudes of all Places, I have taught in the preceding Chapter; and how to find their Longitudes, or the Differences of *Meridians* 'twixt them and any Place aſſignable, ſhall be ſhewn in the ſucceeding part of this Chapter. In the mean time, it may not be amiſs to take notice in this Place, That as the Knowledge of the Longitude and Latitude of any Place is requiſite, before we can point out that Place upon any general *Map*; ſo, till we know theſe Things accurately of all the Remarkable Places upon the Surface of the *Globe*, our general *Maps* of the *World* muſt be incomplete; and as we are hitherto very much wanting in theſe Particulars, our general *Mappography* muſt accordingly be very inaccurate. And whereas if we keep our *Eye* always in the ſame Pole of the Plane of that *Meridian* upon which we project the *Sphere*, and with the *Eye* in this Poſition delineate the Remarkables on the Surface of the *Earth* upon that Plane; the *Hemiſphere* made by this Plane next to the *Eye* will appear monſtrouſly diſproportionate to that which is oppoſite to it, in the Plane of the Projection, and that the more ſo, the nearer the Places to be projected to the *Eye*, ſo that ſuch Places as are very near it are projected at an immenſe Diſtance from the Centre of the Projection. The *Geographers*, to remedy this Inconveniency, generally project one *Hemiſphere* of the *Globe* with the *Eye* in one of the Poles of the *great* Circle whoſe Plane they project it upon, and the other with the *Eye* in the op-

poſit

posite Pole of the same *great* Circle, *viz.* the opposite *Hemispheres* with the *Eye* in the opposite Poles of the *great* Circle upon whose Plane the Projection is drawn; by which means the Whole Surface of the *Earth* falls within the *Primitive* Circle; and is much more easily delineated, and the Situations of its Parts with respect to each other are much better apprehended, than if they were to keep the *Eye* wholly in one *Pole* of the *great* Circle upon whose Plane they draw their Projection, and thereby were to run out in the Delineation of those Parts of the *Globe* near the Point where the *Eye* is plac'd to an almost infinite Distance from the Centre of the Projection. Indeed, when little more than one *Hemisphere* of the *Globe* is represented in a *Map*, they keep their *Eye* in the same Pole of the *great* Circle upon whose Plane the Projection is drawn; but never in *Maps* of the *World*, by reason of the aforesaid Inconveniency: The whole *Sphere* of our *Earth* being always by them divided into two *Hemispheres*, of which the one may be conceiv'd as represented on one side, and the other on the other side of the Plane of that *Meridian* which we make choice of to project the *Sphere* upon. And the same Method is observ'd if they draw a general *Map* of the *World* upon the Plane of the *Equator*; which is often done, in order to give us a more accurate Idea of the Situation of the *Circumpolar* Parts of the *Earth*.

Schol. 2. After the same manner as we discover the Points in the Plane of the Projection where any Places upon the Surface of the *Earth* will fall from their Longitude and Latitude given, we may find out like Points in a like Projection of the *Sphere* where any *Fix'd Stars* will fall, if we have their Right Ascension and Declination;

or by their Longitude and Latitude we may do the same; as every one must see, who is Master of this *Problem*, or of the preceding Chapter. So that general *Maps* of the *Heavens* are drawn after the same manner as general *Maps* of the *Earth*. And for the former, we are furnished with much better Materials than for the latter: For whereas in *Geography* we are acquainted with the Longitudes and Latitudes of; comparatively, but a few of the Remarkable Places upon the Surface of the *Earth*, and amongst those which we are acquainted with, a great many are not well known with sufficient Accuracy, which makes our *Maps* and *Globes* of the *Earth* so incomplete as they are at present; in *Astronomy* the Case is quite otherwise; the greatest of *Astronomers*, our excellent Mr. *Flamstead*, having by incredible Industry and Accuracy determin'd the Right Ascensions and Declinations, Longitudes and Latitudes, of all the Remarkable *Fixed Stars* which appear above our *Horizon*, from the best *Observations* that ever were made; and the ingenious Mr. *Halley*, having given us a *Catalogue* of the Places of those which never appear to us, by reason of their Nearness to the *Southern* Pole: Which last *Catalogue*, if done with like Accuracy and Exactness, may be look'd upon as a Supplement to Mr. *Flamstead*'s: And from both together we may easily frame perfect *Celestial Globes* instead of those erroneous ones which we have at present, and such general *Maps* of the *Heavens* as the World has not hitherto been provided with.

Schol. 3. If we have a mind to draw *Maps* of particular large *Countries*, or of any Number of *Constellations* in the *Heavens*, they may be done after the same manner with the general *Maps*; that *Hemisphere* being projected within which they

they fall, and one of the Extremities of thoſe Countries, or the Firſt *Fix'd Star* in thoſe *Conſtellations*, being laid down according to their Latitude and Right Aſcenſion (for Inſtance) in the Limb of the Plane of the Projection: For having the Differences of the *Meridians* 'twixt all the Remarkable Places in thoſe Countries and that *firſt* laid down; and the Differences of Right Aſcenſions of all the *Stars* in thoſe *Conſtellations*; we may eaſily project 'em, after the ſame manner as we find the Place of *Vienna* in the preſent *Problem*. And thus may we project any part of the *Earth*, or the *Heavens*, upon the Planes of any other of the *Great* Circles of the *Sphere*, as well as upon what I have inſtanced in; in like manner as we can ſolve the *Problem* before us upon the Plane of any other *great* Circle of the *Sphere*, as well as upon that of the *Meridian*. But this the Reader, who underſtands *Prob.* IX. *Cap.* III. with its Solutions, muſt be ſufficiently ſenſible of.

Schol. 4. As to the Methods of drawing *Maps* of particular ſmall Countries, they do not come directly under my Conſideration in this Place, as they do not depend upon a Reſult from the Projection of the *Sphere in Plano*: For every ſmall Country is taken by the *Mappographers* for a Plane, as that Part of the Surface of our *Earth* which it comprizes differs very little from one; and that the leſs, the ſmaller the Country to be deſcrib'd. And having the Length, Breadth, Figure, *&c.* of a Plane Country, and the Bearings of its Remarkable Places from each other, with their Diſtances, *&c.* it is eaſy to repreſent them in what Compaſs we pleaſe, according to what Scale we make choice of; by 18 VI. *Euclid*. But upon this Subject, the Reader, who is unac-

quainted with these Matters, may consult *Vareni Geograph. pag.* 466, 467, *&c. Edit. Newtonian.* where he will also find General *Mappography* treated at large before the particular Part. What has been said here upon general *Mappography*, together with a thorough Understanding of the *Second* Chapter of this Book, I look upon as sufficient to enable any one to draw general *Maps* of the *Earth*, or of the *Heavens*, upon the Plane of any one of the *great* Circles of the *Sphere*, as also *Maps* of particular large *Countries*, or *Constellations*: And upon what Planes they are represented the best, tho' I have sufficiently hinted, I had rather the Reader should discover upon his own Trial; because he will then not only be acquainted with the thing, but with the Reason of it at the same Time.

PROBLEM II.

Having any number of Places *projected upon a General* Map *of the World; to find out the Latitudes of those* Places *and their Difference of Longitude one from another.*

SOLUTION.

LET the two Places refer'd to in the preceding *Problem, viz. London* and *Vienna*, be those whose Latitudes and Difference of Longitude are requir'd; the one being found projected in the Point *L*, and the other at *v*: 'Tis plain, that
Fig. 30. the Place of *London* being in the Limb of the Projection, in this Case the *Sphere* is projected upon the Plane of its *Meridian*; and if we open our *Sector* to the *Radius* of our Projection, and taking the Arch ♎ *L* betwixt our Compasses, measure it upon the Line of *Chords*, we shall find the

the Latitude of *London*, in this Projection, to be 51° 32′ 00″. And if thro' the Place of *Vienna* at v, and the two Poles of the World p and π, we strike a *great* Circle, by 5. IV. *Euclid*, it will be the Circle of Longitude of *Vienna*, having q for its Centre; and $\alpha\, q$ being measured upon the Line of Tangents will give us the *Inclination* of the Plane of the Circle of Longitude of *Vienna* to the Plane of the Projection, or the Difference of Longitude 'twixt *London* and *Vienna*, 17° 07′ 30″; by *Coroll. ad Prop.* III. *Sect.* II. *Cap.* II. And finding the Pole of the Circle of Longitude of *Vienna* within the Plane of the Projection to fall upon the Point g, by *Lemma* I. *Sect.* III. *Cap.* II. a Line drawn from g through v to the Limb of the Projection, will cut it in x making x ♎ equal to the *North* Latitude of *Vienna*; by *Coroll.* 1. *ad Lemma* II. *Sect.* III. *Cap.* III. which measur'd upon the Line of *Chords*, gives us the Latitude of *Vienna*, 48° 22′ 00″.

Q. E. F.

Scholium. This *Problem* is the Converse of the preceding: And after the same manner may we find out the Latitudes, and Difference of Longitude, of any other Places of a general *Map* of the *World*, the *Meridian* of whatever Place upon the *Earth* the *Mappographer* pitches upon for his *First Meridian*, or that upon whose Plane the Projection is drawn: For having their Places in the *Maps*, we may find out their Circles of Longitude, as we do that of *Vienna*; and having those, we may find out the Inclinations of the Planes of those Circles to that of the Projection; or the Difference betwixt theirs and that *First Meridian* which the *Mappographer* makes choice of: And having their *Inclinations* to the Plane of the Projection, we may find their Poles which fall within that

that Plane; and thence their respective Latitudes. And in General *Maps* of the *Heavens*, after the very same manner, we may find out the Right Ascensions and Declinations of any *Stars* drawn in them; as every one must see, without the help of particular Instances. In *Maps* of particular Large *Countries* or *Celestial Constellations*, drawn after the same manner, the Method of Proceeding is the same: For completing any two or more of the *Meridians* drawn in them, (if they are not already completed in the Projection) which is done by finding out their Centres, by 5. IV. *Euclid*, and bissecting a Right Line drawn to their opposite Points of Intersection, we have the Centre of that *Meridian* on whose Plane the *Maps* are drawn; from which Centre to each Pole, or Point of Intersection, is its *Radius*; which *Meridian* with these *Data* may easily be drawn: Which being got, we proceed, as in General *Maps* of the *Earth* or the *Heavens*, in investigating the Longitudes and Latitudes, and Right Ascensions and Declinations, of such *Places* upon *Earth*, and such *Stars* in the *Heavens*, as are mark'd out upon those particular *Maps*. The *Reader* may only take Notice, That in *Maps* of particular *Countries*, 'tis requisite that he know the *First Meridian* which the *Mappographer* makes choice of, which he is generally advised of by the *Artist*, or which he may find out by the Longitude ascrib'd to the most *Westerly Place* in the *Map*, or any other Place in it whose *Longitude* is mark'd out, and by enquiring what places upon the *Globe* have precisely so much less *Longitude* than, or have their *Meridians* precisely so much *Westerly* of, those Places in the *Maps*. In *Maps* of particular *Constellations*, the Right Ascensions of all *Stars* are reckon'd from the *First* Point of *Aries*. If a *Map* of the *World*, or of the *Heavens*, &c. is projected upon any other

other *great* Circle of the *Sphere*; the Solution of the *Converse* of the preceding *Problem* upon that Projection, will be the Solution of this *Problem* upon the same: Which whoever understands the II^d and III^d Chapters of this Book, particularly the *last Problem* of the III^d Chapter, must be able to do, without any further Illustration.

POSTULATUM.

The Distance 'twixt two Places upon the Earth, *is measur'd by an Arch of a Great Circle passing thro' the Places propos'd.*

PROBLEM III.

Having two or more Places projected upon a general Map *of the World, and thence the Latitudes of those Places, and their Difference of Longitudes one from another; to find out the Distance of those Places from each other in* Geometrical *Miles, assuming* 60 *such Miles to be equal to* One Degree *of a* great *Circle of the* Earth's *Globe.*

SOLUTION *upon the Projection.*

LET the two Places refer'd to in the preceding *Problem*, viz. *London* and *Vienna*, be those whose Distance is requir'd; *London* being found projected in the Point *L*, and *Vienna* in the Point *v*. Now, because the Point *L*, representing *London*, is in the Periphery of the Plane of Projection thro' it and the Centre *a*, draw the Diameter *L a μ*, and a *great* Circle thro' the Points *L*, *v*, and *μ*, by the 5. of IV. of *Euclid*; and find *Fig.* 30.

find its Pole *y*, by *Lemma* I. *Sect.* III. *Cap.* II. a Line drawn from *y* thro' *v* to the Limb of the Projection will cut it in *z*, making *z L* equal to the Distance 'twixt *London* and *Vienna* 11° 32′ 12″ equal to 692½ *Geometrical* Miles.

Coroll. After the same manner may the Distance of any other two Places projected in the General *Map* be found, *viz.* by finding the Pole of the *great Circle* passing thro the Places, and laying a Ruler over the *Pole* and the two Places, and the *Arch* cut off in the Limb of the Projection, will shew the Distance requir'd.

Trigonometrical SOLUTION.

Fig. 37. IN *Fig.* 37. let *p* represent the *North Pole*, *L* the Place of *London*, *V* that of *Vienna*, *p L* an Arch of the *Meridian* passing thro' *London*, *p V* the Arch of the *Meridian* of *Vienna* contain'd between the Pole *p*, and *V* the Place of *Vienna*, *L V* an Arch of a great Circle passing thro' *London* and *Vienna*; then in the oblique-angl'd Spherical Triangle *p L V* are given *p L* the Complement of the Latitude of *London*, or its Distance from the Pole 38° 28′ 00″; *p V* the Complement of the Latitude of *Vienna*, or its Distance from the Pole 41′ 38′ 00″; and the Angle *L p V* the Inclination of the Planes of the Meridians passing thro' the two Places, which is equal to their Difference of Longitudes 17° 07′ 30″; to find the Length of the Arch *L V*, the Distance requir'd.

Having let fall the Perpendicular *Lr* in the Right-angl'd Spherical Triangle *L p r*, are given *L p* the Hypothenuse 38° 28′ 00″, the Angle *p* 17° 07′ 30″, whence *p r* may be found, by the following Proportion.

R : c.S.

$R : c.S.\ Lpr :: T.\ Lp : T.pr$

Now, $pV - pr = rV$

Again, $c.S.\ pr : c.S.\ pL :: c.S.\ Vr : c.S.\ VL$

$R : c.S.\ Lpr$	17° 07′ 30″	9. 980305
$:: T.Lp$	38 28 00	9. 900086
$: T.\ pr$	37 12 29	9. 880391

$pV = 41°\ 38'\ 0'' - pr = 37°\ 12'\ 29'' = rV\ 4°\ 25'\ 31''$

$c.S.\ pr$	37° 12′ 29″	9. 901153
$: c.S.\ pL$	38 28 00	9. 893745
$:: c.S.\ Vr$	04 25 31	9. 998703
$: c.S.\ VL$	11 26 00	9. 991295

Whence the Diſtance between *London* and *Vienna* is 11° 26′ 00″ equal to 686 *Geographical* Miles.

Case 2[d]. If the Places propos'd had been under the ſame Meridian, or, which is the ſame thing, had been in the ſame Circle of Longitude; then the Difference of their Latitudes had been their Diſtance: For the Arch of the *great* Circle paſſing thro' the two Places, is the Arch of Difference of Latitude. Thus, ſuppoſe it were requir'd to find the Diſtance 'twixt *Vienna* in *Auſtria*, and *Mount Ætna* in the Kingdom of *Sicily*, becauſe theſe two Places lie under the ſame Circle of Longitude; from the Latitude of *Vienna* 48° 22′ 00″ take the Latitude of *Mount Ætna* 38° 20′ 00″, the Remainder 10° 02′ 00″ is the true Diſtance 'twixt theſe two Places, equal to 602 *Geographical* Miles.

Case 3[d]. If the Places propos'd lie under the *Equator*; then the Difference of Longitude is

is equal to their Distance. Thus suppose it were requir'd to find the Distance 'twixt the *Mouth of the River Amazones* on the *Eastern Coast* of *America*, and *Cape Passage* on the *Western Side* of the same *Coast*, both which Places lie under the *Equator*, and which will determine the Breadth of that Part of *America* which lies under the *Equator*.

From the Longitude of the *Mouth* of the *River Amazones* 309° 30′ 00″ take the Longitude of *Cape Passage* 275° 50′ 00″, the Remainder 33° 40′ 00″ equal to 2020 *Geographical* Miles, is the Distance 'twixt those two Places, or the Breadth of *Amreica* in that Part.

N. B. The Longitude of the two Places in this last *Case*, are reckon'd from the Meridian of *London*.

CASE 4. If one of the Places be projected under the *Equator* and the other at some Distance to the *Northward* or *Southward* of the same; the Method for finding the Distance will be more troublesome than in the last *Case*, but not so complicate as in the first. For *Example:*

Let it be requir'd to find the Distance 'twixt *London* in the *North* Latitude of 51° 32′ 00″, and the *Mouth* of the *River Amazones* lying under the *Equator*, and whose Difference of Longitude, or Angle which the Planes of their Meridians form with each other, is 50° 30′ 00″.

The *Mouth* of the *River Amazones* is projected to the Westward of *London* upon the *Globe* of the Fig. 38. *Earth:* therefore in *Fig.* 38. let *L* represent the Place of *London*, *A* the *Mouth* of the *River Amazones*, *p* the North Pole, *p* ♎ the Meridian of *London*, *p A* the Meridian of the *Mouth* of the *River Amazones*, *A* ♎ the Arch of the *Equator* intercepted 'twixt the two Places equal to the Angle *A p* ♎ the Difference of their Longitudes, and *A L* the Distance of the two Places.

Now

Now because *A* ♎ is an Arch of the *Equator* of which *p* is the Pole; therefore the Angle *p* ♎ *A* is a Right Angle: Wherefore, in the Spherical Triangle, *AL* ♎ Right-angl'd at ♎, are given L ♎ the Latitude of *London* 51° 32′ 00″, *A* ♎ the Difference of the Longitudes of *London* and the *Mouth* of the *River Amazones* 50° 30′ 00″, and the Angle at ♎ Right; whence to find *AL* the Distance, the Proportion will be,

R : *c.S.* *L* ♎ :: *c.S.* *A* ♎ : *c.S.* *A L*.

R : *c.S.* *L* ♎	51°	32′	00″	9. 793832
:: *c.S.* *A* ♎	50	30	00	9. 803510
: *c.S.* *L A*	66	41	30	9. 597342

Whence the Distance 'twixt *London* and the *Mouth* of the *River Amazones* is 66° 41′ 30″ equal to $4001\frac{1}{2}$ *Geographical* Miles.

Case 5th. If the Places propos'd are not situated under the *Equator*, as in *Case* 3. nor under the same Meridian, as in *Case* 2. but under the same Parallel; or, which is the same, If their Distances from the Pole are equal: as suppose the *City* of *Liege*, whose Latitude is 50° 40′ 00″, and *Prague* the Capital City of *Bohemia*, whose Latitude also is 50° 40′ 00″, and their Difference of Longitude 08° 30′ 00″, *Liege* being so much to the Westward of *Prague*, and it were required to find the Distance of those two Places; Then, in *Fig.* 39. let *e* represent the Place of *Liege*, *g* that of *Prague*, *p* the *North Pole*, and *Fig.* 39.
e g the *Arch of Distance*: Wherefore having let fall the Perpendicular *p r*, the Triangles *e p r*, and *g p r* are equal; and to find *e r*, in the Triangle *e p r*, equal to half *e g* the Distance, are given *p e* the Complement of the common Latitude or Distance of

of either of the Places from the Pole 39° 20′ 00″ the Angle *epr* equal to half the Difference of their Longitudes 04° 15′ 00″, and the Angle at *r* Right: whence to find *er*, the Proportion will be,

$$R : S.\, epr :: S.\, ep : S.\, er$$

$R : S.\, epr =$ 04°	15′	00″	8. 869868
$:: S.\, ep =$ 39	20	00	9. 801973
$: S.\, er =$ 02	41	32	8. 671841

The Double of which is 05° 23′ 04″, equal to 323$\frac{2}{15}$ *Geographical* Miles, the Distance 'twixt the two Places.

Defin. The Angle of Position is the same with the Angle form'd by the Plane of the Meridian passing through one of the Places, and the Plane of the *great* Circle passing through the two Places, and is measur'd by an Arch of a *great* Circle describ'd about one of the Places proposed, and intercepted 'twixt the Meridian passing thro' it and the *great* Circle passing over the two Places.

Fig. 37. Thus, in *Fig.* 37. the Angles of Position 'twixt *London* and *Vienna*; is the Angle *pLV*; but the Angle of Position 'twixt *Vienna* and *London* is the Angle *pVL*.

Let it be requir'd to find the Angle of Position 'twixt *London* and *Vienna*; *London* being found projected in *Fig.* 30. in *L*, and *Vienna* in the same Projection in the Point *v*.

Having found *Lv* the Distance of *London* from *Vienna*, in the preceding *Problem*, to be 11°. 32′ 12″, it will be,

S. LV

S. LV : S. LpV :: S. pV : S. pLV

S. LV =	11°	26′	00″	9. 297164
: *S. LpV* =	17	07	30	9. 469022
:: *S. pV* =	41	38	00	9. 822104
: *S. pLV* =	80	42	33	9. 994262

Which taken from 180° 00′ 00″, (becauſe the Angle *pLV* is Obtuſe) leaves the Angle of Poſition 'twixt *London* and *Vienna* 99° 17′ 27″.

If the Angle of Poſition 'twixt *Vienna* and *London* be requir'd, then the Proportion will be,

S. LV : S. LpV :: S. pL : S. pVL

S. LV =	11°	26′	00″	9. 297164
: *S. LpV* =	17	07	30	9. 469022
:: *S. pL* =	38	28	00	9. 793832
: *S. pVL* =	67	31	26	9. 965690

Whence the Angle of Poſition 'twixt *Vienna* and *London* is 67°. 31′. 26″. So that, according to the Nautical way of ſpeaking, if *Vienna* bears from *London* South 80° 42′ 33″ Eaſt, or E. by S. 01° 57′ 33″ E. *London* bears from *Vienna* North 67° 31′ 26″ Weſt, or W. N. W. 0° 1′ 26″ W. Whence the Difference of the oppoſite Rhumbs is 13° 31′ 07″.

The Angle of Poſition might have been found without the Diſtance, by the help of the Latitudes of both Places, and their Difference of Longitude.

For having found *pr*, and thence *rv*, as in the *Firſt Caſe* of this *Problem*; to find the Angle *LVr*, the Proportion will be. Fig. 37.

O *S. Vr*

$S.Vr : S.pr :: T.Lpr : T.LVr.$

$S.\ Vr =$	04°	25′	30″	8. 887385
$:S.pr =$	37	11	00	9. 781301
$::T.Lpr =$	17	07	30	9. 488717
$:T.LVr =$	67	29	36	10. 382633

Differing but 50″ from the former Calculation.

In *Case* 2. where the Places proposed were situated under the same *Meridian*, the Angle of Position vanished: For the lesser the Difference of Longitude or Inclination of the Planes of the Meridians passing through the two Places is, the lesser is the Angle of Position between that Place which is most remote from the Pole, and that which is nearest; till at last when the Difference of Longitude vanishes, the Angle of Position vanishes also.

In *Case* 3. where the Places proposed lie under the *Equator*, the Angle of Position is equal to 90 *Deg.* For the Distance of the *Pole* from the *Equator* being every way equal to a Quadrant, the Inclination of the two Planes to each other must be equal to a Quadrant also.

In *Case* 4. where one of the Places proposed is under the *Equator*, and the other in *North Latitude*, the Angle of Position is easily had by one Proportion.

Fig. 38. For in *Fig.* 38. where *L* represents *London*, and *A* the *Mouth* of the *River Amazones*, the Angle of Position 'twixt *London* and the *Mouth* of the said *River Amazones* is easily had: For,

: : S.

∴ S. L ♎ : R :: T. A ♎ : T. A L ♎

S. L ♎ =	51° 32′ 00″	9. 893745 —
∴ R :: T. A ♎ =	50 30 00	10. 083895
T. A L ♎ =	57 09 38	10. 190150

If the Angle of Position 'twixt the *Mouth* of the *River Amazones* and *London* be requir'd; then, by the Laws of *Spherical Triangles*, the Proportion will be,

S. A ♎ : R :: T. L ♎ : T. L A ♎

S. A ♎ =	50° 30′ 00″	9. 887406 —
: R :: T. L ♎ =	51 32 00	10. 099913
: T. L A ♎ =	58 29 23	10. 212507

Whence it appears, That the Angle of Position 'twixt *London* and the *Mouth* of the *River Amazones* is South 57° 09′ 38″ West; but 'twixt the *Mouth* of the *River Amazones* and *London* is North 31° 30′ 37″ East.

In *Case* 5. where the two Places were projected at equal Distances from the *Pole*, viz. *Liege*, and *Prague* in *Bohemia*, the Latitude of each Place being 50° 40′ 00″, and the Difference of their Longitude, or Angle form'd by the Planes of the two *Meridians* 04° 15′ 00″, the Angle of Position is easily had, by the following Proportion: Fig. 39.

R : c.S. e p :: T. e p r : c.T. p e r

R : c.S e p =	39° 20′ 00″	9. 888444
:: T. e p r =	02 07 30	8. 569432
: c.T. p e r =	88 21 22	8. 457876

 Which

Which is equal to the Angle *e g p*; becauſe *e p* and *p g* are equal: Whence the Agle of Poſition 'twixt either of the two Places is 88° 21′ 22″; that is, 'twixt *Liege* and *Prague* North 88° 21′ 22″ Eaſt, but 'twixt *Prague* and *Liege* North 88° 21′ 22″ Weſt.

Coroll. Hence we ſee the Reaſon of that *Geographical Paradox*, That if *Antwerp* be due *Eaſt* of *London*, for that Reaſon *London* cannot be due *Weſt* of *Antwerp*: For the *Eaſt* and *Weſt* Line, and conſequently all the other *Rhumb* Lines, being dependent upon the *Meridian* or *North* and *South* Lines, and always forming Equal Angles with it; and ſince all the *Meridians*, or *North* and *South* Lines, to which the *Needle* is always conformable, and therefore lies in the ſame Plane, (excluding the Conſideration of Variation, which, in this Caſe, may be totally neglected) meet in the *Poles*, the *Eaſt* or *Weſt* Lines, which form Right Angles with theſe, will meet and interſect each other alſo in given Points; and conſequently, that Place which appears due *Eaſt* in one *Meridian*, from a given Place in another *Meridian*, will be farther diſtant from the *Pole* than the Place firſt given; and conſequently, if we remove to the ſecond Place, the Place firſt given being nearer to the *Pole* than the ſecond, cannot appear due *Weſt*, but conſiderably to the *Northward*, and ſo much the more, by how much the Diſtance of the Two Places is greater: As will evidently appear to any One who will but be at the Pains to draw Two *Meridian* Lines interſecting each other in a given Point, and draw Two Lines at Right Angles to them at the ſame Diſtance from their Interſection or *Pole*. Now, if we travel according to the Direction of the *Eaſt* Line of the *Compaſs*, we ſhall not arrive at the Place in the ſecond *Meridian* that appeared due *Eaſt*, but at another as far diſtant

distant from the *Pole* as the first Place was: And thus in making one entire Revolution, we shall arrive at the same Place we set down from, always keeping at the same Distance from the *Pole*, and describing that Path which is represented upon the *Globe* by the *Parallel of Latitude* passing through the Place first given.

There are other Methods, less correct, for Measuring the Distances of Places from the former *Data*, upon General *Maps*, as also for Investigating the same; but if the *Reader* is well apprised of what has been delivered in this and the former Sections relating to *Projections*, he will easily supply them himself.

CASE 2[d]. If the *Latitudes* of the Two Places were given, and the *Angle of Position* between them, viz. Suppose the *Latitude* of *London* were given 51° 32′ 00″ of *Vienna* 48° 22′ 00″, and the *Angle* of *Position* 'twixt *London* and *Vienna* 99° 17′ 27″; To find their Distance? Then, in the *Spherical Triangle* *p L V* are given *p L*, *p V*, and the *Angle L*: Whence, to find *L V*, by the Laws of *Spherical Triangles*, it will be, *Fig.* 37.

R : *c.S.* ∟ *L* : : *T.p L* : *T.* 4th *Arch.*

And again,

c.S.p L : *c.S. p V* : : *c.S.* 4th *Arch* : *c.S.* 5th *Arch.*

Now 4th *Arch* — 5th *Arch*, gives the Distance.

CASE 3[d]. Again, if the *Latitude* of one of the Places be given, its *Difference* of *Longitude* from another, and the *Angle* of *Position*; To find the *Distance* and *Latitude* of the other? *viz.* Suppose the Latitude of *London* 51° 32′ 00″ be given, its Difference of Longitude from *Vienna* 17° 07′ 30″

West, and the Angle of Position 'twixt *London* and *Vienna* 99° 17′ 27″; then, in the Oblique-angl'd Spherical Triangle *L p V* are given *p L*, the Complement of the Latitude of *London*, the Angle *p* the Difference of Longitude 'twixt *London* and *Vienna*, and Anlge *L* the Angle of Position: Whence to find *L V* the Distance, it will be, by the known Laws of Spherical Triangles,

Fig. 37.

R : *c.S.* ∟ *p* : : *T.* ∟ *p* : *t.T.* a 4th *Arch*,
The ∟ *L* — 4th *Arch* gives a 5th *Arch*.

Again

c.S. 4th *Arch* : *c.S.* 5th *Arch* : : *T.* ∟ *p* : *T. L V*.
For the *Latitude*, it will be,
S. ∟ *p* : *S. L V* : : *S.* ∟ *L* : *S. p V*,
or *c.S.* of the *Latitude*.

Case 4th. Again, If the *Difference of Longitude*, or Inclinations of the Planes of the *Meridians*, passing through the Places, be given, together with the *Angles of Position* of each Place to the other respectively; then, in the *Spherical Triangle L p V*, are given the *Angles L*, *p* and *V*: Whence may be found the *Side L V* the Distance, or the *Sides p L* and *p V*, the Complements of the *Latitudes*, by the following Proportions.

Fig. 37.

R : *c.S.* one of the Angles of Position : : So is the Co-sine of the other Angle of Position to a 4th Sine. As the 4th Sine to the Sine of ½ the Sum of the 3 Angles, (the Supplement of the Obtuse Angle being first obtain'd, and used instead of the Obtuse Angle;) so is the Sine of the half Sum lessened by the Difference of *Longitude* to a 7th Sine: To which if the *Radius* be added, half that Sum will be

be the Co-sine of half the Distance requir'd: And for the Latitude, it will be,

S. ∟ *p*: *LV*: : S. ∟ *L*: *c.S.* of the *Latitude* of the Place represented by *V*; and so is the *Sine* of the ∟ *V* to the *c.S.* of the *Latitude* of the Place represented by *L*.

CASE 5th. If the Distance of two Places lying under the same *Meridian*, viz. *r* and *V* and the Angles of position *r* and *V* of the Places *r* and *V* to an unknown Place *L*, were given; To find the Distances of the Place *L* from the Places *r* and *V*? In the Spherical Triangle *L r V* are given the Side *V r*, the Difference of Latitude of the Places *r* and *V*, and the adjacent Angles *r* and *V*: Whence to find *L r* and *r V*, the Proportions will be, Fig. 37.

R: *c.S. r V*: : *T.* ∟ *V*: *T.* 4th *Arch*:

∟ *r* — 4th *Arch* gives a 5th *Arch*, if the Angle *r* be greater than the 4th *Arch*; otherwise the 4th *Arch* — ∟ *r* gives a 5th *Arch*.

Again, *c.S.* 4th *Arch*: *c.S.* 5th *Arch*: : *T. r V*: *T. L r*; and for *L V*, it will be,

S. ∟ *V*: *S. L r*: : *S.* ∟ *r*: *S. L V*.

Several other *Cases* relating to Distances might be proposed, but the intelligent *Reader* will easily supply the rest himself.

PROBLEM IV.

Having the Distances of an Unknown Place from Two Places projected upon a General Map of the World: *Whence the* Latitudes *of those Two Places, and their* Difference of *Longitude one from another, are given, to find out the* Point *representing the Unknown Place upon the General* Map; *and thence its* Latitude, *and* Difference of Longitude *from either of the Known Places.*

SOLUTION *upon the Projection.*

Fig. 30. LET the Two Places refer'd to in the preceding *Problems*, viz. *London* and *Vienna*, be the Two Places whose *Latitudes* and *Difference of Longitude* one from the other are given, *London* being found projected in the Point *L*, and *Vienna*, in the Point *v*; and let *Rome* be the Place whose Distance from each of the two former Places is given, *viz.* from *London* 13° 09° 48″ Equal to 789$\frac{4}{5}$ Geometrical Miles; and from *Vienna* 07° 06′ 40″ Equal to 426$\frac{2}{3}$ Geometrical Miles, and whose Place upon the General *Map* is requir'd, and thence its *Latitude* and *Difference of Longitude* from either of the Two given Places, *viz. London* or *Vienna*. Now because the Point *L*, representing *London*, is found projected in the Periphery of the Plane of Projection, about that Point *L* describe a Parallel, or small Circle, Equal to 13° 09′ 48″, or 789$\frac{4}{5}$ Geometrical Miles, by *Cap.* II. *Sect.* IV. *Prop* II. And because the Point *v*, representing *Vienna*, is projected within the Periphery of the Plane of Projection, about the Point *v*, describe a Parallel or small Circle Equal to 07° 06′ 40″, or 426$\frac{2}{3}$ Geometrical Miles, by

Cap.

Cap. II. *Sect.* II. *Prop.* IV. the Interſection of theſe Two Circles in the Point *R* will be the Place of *Rome*, in this *Projection*. And if through the Points *p*, *R* and *π*, a Circle be drawn, by the 5th of the IV, of *Euclid*, it will repreſent the Meridian of *Rome*; Whence its Latitude or Meaſure of the Arch *p R*, and the Inclination of the Planes of the Circles *p L*, and *p R*, or *p R* and *p v*, the Difference of Longitude of *London* and *Rome*, or of *Rome* and *Vienna*, may be obtain'd by *Prob.* II. *Sect.* I. *Cap.* IV.

Coroll. After the ſame manner may Points be found upon the Plane of Projection, repreſenting any Number of Places, provided the Diſtance of each Place from two known Places, already projected upon the *Map*, be had, and thence the Latitudes of each, and their Differences of Longitude one from the other: But inaſmuch as the Parallels, or ſmall Circles, cut each other in two Points; in order to make a right Choice of the two Points thus found, Regard muſt be had to the Bearing or Angle of Poſition of one of the two Known Places to the Unknown, and then the Thing is eaſily done.

Scholium. Tho' this Method, of finding out Points repreſenting the ſeveral Places upon the *Earth*, and thence drawing General *Maps of the World*, be not ſo applicable to *Geography*, by reaſon of the great Difficulty there is in finding out and determining the true Diſtances of Places from each other; yet, in the Delineating of *Maps of the Conſtellations*, it may be of great Uſe: Becauſe the mutual Diſtances of the Stars are obtain'd to the greateſt Exactneſs that the Nature of Inſtruments will allow of, and are the Principal *Data* by which *Aſtronomers* determine their *Lati-*

tudes, *Longitudes*, *Right Ascensions* and *Declinations*; and, by the help of this, the *Path* of a *Comet* or *Planet* may be trac'd out upon the Surface of a *Globe*, or upon a General *Map of the Constellations*, without any previous Calculation.

Trigonometrical SOLUTION.

Fig. 40. IN *Fig.* 40. which we will refer to, to avoid Confusion, let *p* represent the North-Pole, *L* the Place of *London*, *V* that of *Vienna*, and *R* that of *Rome*. *P L* the Arch of the Meridian passing through *London*, equal to 38° 28′ 00″ the Complement of its Latitude. *P V* an Arch of the Meridian of *Vienna*, equal to 41° 38′ 00″ the Complement of its Latitude. *L R* an Arch of a Great Circle passing through *London* and *Rome*, equal to their mutual Distance, which we here assume 13° 09′ 48″ equal to 789 4/5 Geometrical Miles, and *R V* an Arch of a great Circle passing through *Rome* and *Vienna* which we also assume here 07° 06′ 40″ equal to 426 2/3 Geometrical Miles; To find *P L* the Latitude of *Rome*, and the Angle *L P R* or *R P V* the Inclinations of the several Meridians, or the Differences of Longitude 'twixt *London* and *Rome*, or *Rome* and *Vienna*.

Through *L* and *V* let the Arch of a great Circle be drawn, representing the mutual Distance 'twixt *London* and *Vienna*; then in the Oblique-angl'd Spherical Triangle *P L V* are given *P L* 38° 28′ 00″, *P V* 41° 38′ 00″, and the Angle *V P L* the Difference of Longitude 'twixt *London* and *Vienna*, which we here assume 17° 07′ 30″; To find *L V* their mutual Distance, and the Angle *P L V* the Angle of Position 'twixt *London* and *Vienna*:

It

It will be, by the Laws of *Trigonometry*,

R : *c.S. L p V* : : *T. L p* : *T.* 4th *Arch*;

Now *p V*—4th *Arch* gives a 5th *Arch*.

Again,

c.S. 4th *Arch* : *c.S. p L* : : *c.S.* 5th *Arch* : *c.S. V L*.

R : *c.S. L p r*	17° 07′ 30″	9. 980305
: : *T. L p*	38 28 00	9. 900086
: *T.* 4th *Arch*	37 12 29	9. 880391

$pV = 41° 38' 00'' - 37° 12' 29'' = 04° 25' 31''$

c.S. 4th *Arch*	37° 12′ 29″	9. 901153
: *c.S. P L*	38 28 00	9. 893745
: : *c.S.* 5th *Arch*	04 25 31	9. 998703
: *c.S. L V*	11 26 00	9. 991295

For the *Angle p L V*, it will be,

S. L V : *S. p V* : : *S. L p V* : *S. p L V*

S. L V	11° 26′ 00″	9. 297184
: *S. p V*	41 38 00	9. 822404
: : *S. L p V*	17 07 30	9. 469022
: *S. p L V* 80	42 33	9. 994262

Whose Supplement to 180° 00′ 00″, *viz.* 99° 17′ 27″, is the *Angle* requir'd.

2*dly*, In the Oblique-angled Spherical Triangle *L V R*, are given *L V* the Distance 'twixt *London* and *Vienna* 11° 26′ 00″; *L R* the Distance 'twixt *London* and *Rome* 13° 09′ 48″; and *R V* the Distance

Distance 'twixt *Rome* and *Vienna* 07° 06′ 40″: Whence to find the *Angle* *R L V*;

To 13° 09′ 48″ = *L R*
Add 11 26 00 = *L V*
And 07 06 40 = *R V*
31 [illegible] 28 = Sum.
15 51 14 = Half Sum.
07 06 40 = *R V* the Side opposite.
08 44 34 = Remainder.

To the *Co-Arith* of the *Sine* of *L R* } = 13° 09′ 48′ 0. 6429083
Add the *Co-Arith* of the *Sine* of *L V* } = 11 26 00 0. 7028359
Also the *Sine* of ½ Sum = 15 51 14 9. 4364570
And the *Sine* of the Remainder } = 08 44 34 9. 1818400

The *Sine* of ½ Sum is 16° 22′ 23″ 19. 9640412
Which doubl'd, gives 32 44 46 9. 9820206
the *Angle* *R L V* requir'd.

To the *Angle* *P L V* = 99° 17′ 27″
Add the *Angle* *R L V* = 32 44 46
Sum is the *Angle* *P L V* = 132 02 13 the *Angle of Position* 'twixt *London* and *Rome*, reckon'd from the *Northern* Part of the *Meridian*.

3*dly*, In the Triangle *p LR* are given *p L* the Complement of the Latitude of *London*, 38° 28′ 00″, *L R* the Distance 'twixt *Rome* and *London* 13° 09′ 48″, and the Angle of Position *p L R* 132° 02′ 13″, or its Supplement 47° 57′ 47″; To find out *p R* the Complement of the Latitude of

Rome,

Rome, and the Angle *L p R* the Difference of Longitude 'twixt *London* and *Rome*: And, by the Laws of the *Spherical Triangles*, it will be,

R : *c.S.*∟ *p L R* : : *T.* *L R* : *T.* 4th *Arch.*

R : *c.S.*	47°	57′	47″	9. 825822
: : *T.*	13	09	48	9. 368979
: *T.*	08	54	01½	9. 194801

Which added to *p L* 38° 28′ 00″, gives 47° 22′ 01″½ a 5th *Arch.*

Again, As *c.S.* 4th *Arch* : *c.S.* 5th *Arch* 47° 22′ 01″ ½ : : *c.S.* *L R* : *c.S.* *p R*

c.S. 4th *Arch*	08°	54′	01″ ½	9. 994730
c.S. 5th *Arch*	47	22	01 ½	9. 830789
c.S. *L R*	13	09	48	9. 988436
c.S. *p R*	48	07	22	9. 824477

Whence the Latitude of *Rome*, by this Investigation, is 41° 52′ 38″ *North*.

For the *Angle* *L p R*, it will be,

S. 5th *Arch* : *S.* 4th *Arch* : : *T* ∟ *p L R* : *T.* ∟ *L p R*.

S. 5th *Arch*	47°	22′	01″ ½	9. 866705
S. 4th *Arch*	08	54	01 ½	9. 189539
T. ∟ *p.L.R*	47	57	47	10. 044999
T. ∟ *L p R*	13	07	47	9. 367833

Whence the Difference of Longitude 'twixt *London* and *Rome*, is here found to be 13° 07′ 47″ to the *Eastward*.

There are various Cases in this *Problem*: As, when the Unknown Place has more or less Longitude, or, which is the same, is situated more to the

the *Eastward* or *Westward* than either of those Places from which the Distances are given, or is situated 'twixt them, as in the preceding Example; or is placed on the same or contrary Sides of the *Equator*, or in the same Meridian with either of the given Places: In which last Case a great part of the Trouble of Calculation is sav'd. But whoever is well acquainted with the former Solution, will find no Difficulty to give ready Answers to any of these, if it shou'd be requir'd.

Fig. 40. CASE 2^d. If the Latitudes and Difference of Longitudes of two Places be given, and their Bearings from a Third, or Angles form'd at the Two given Places, by the Great Circles passing through the Three Places; To find the Latitude of the Third Place, and its Difference of Longitude from either of the given Places.

1st, In the Spherical Triangle *LpV* are given *Lp*, *pV*, and ∟*p*: Whence, to find *LV*, it will be, by the Laws of *Spherical Triangles*,

R : *c.S. LpV* :: *T. Lp* : *T.* 4th *Arch.*

pV — 4th *Arch* gives a 5th *Arch*, if *pV* be greater than the 4th *Arch*; but if *pV* be less, then the 4th *Arch* — *pV* gives a 5th *Arch*.

Again,

c.S. 4th *Arch* : *c.S.* 5th *Arch* :: *c.S. pL* : *c.S. VL*

And for the *Angle* at *L*, it will be,

S. LV : *S.* ∟*p* :: *S. pV* : *S.* ∟*pLV.*

2*dly*,

2*dly*, In the *Triangle* *L R V* are given *L V* and the adjacent *Angles* *L* and *V*; whence to find *L R*, the Proportions will be,

R : *c.S.* *L V* :: *T.* ∟ *V* : *c.T.* 4th *Arch*.

The 4th *Arch* — ∟ *V L R* gives a 5th *Arch*.

Again,

c.S. 4th *Arch* : *c.S.* 5th *Arch* :: *c.T.* *L V* : *c.T.* *L R*.

3*dly*, In the *Triangle* *p L R* are given *p L*, *L R*, and the Angle *p L R*, equal to the Sum of *p L V* and *V L R*; whence to find the *Angle* *L p R*, the Proportion will be,

R : *c.S.* ∟ *p L R* :: *T.* *L R* : *T.* 4th *Arch*.

p L + 4th *Arch*, if the *Angle* *p L R* be Obtuse; but if Acute, *p L* — 4th *Arch* gives a 5th *Arch*; whence to find ∟ *p*, it will be,

S. 5th *Arch* : *S.* 4th *Arch* :: *T.* ∟ *R L p*, *T.* *R p L*, the Difference of Longitude 'twixt the Place *L* and *R*.

Again, For *p R*, the Complement of the Latitude, it will be,

S. ∟ *p* : *S.* *L R* :: *S.* ∟ *L* : *S.* *p R*,

Or,

c.S. 4th *Arch* : *c.S.* 5th *Arch* :: *c.S.* *L R* : *c.S.* *p R*.

Case 3[d]. If the Latitudes of both Places are given, and the Distance between them, to find the Difference of their Longitudes; then, in the *Spherical Triangle* *L p V* are given *L p* and *p V* the Complements of the Latitudes of the Two Places and *L V* their Distance; whence to find ∟ *p* the *Fig.* 37.

∟*p* the Difference of their Longitudes, it will be,

As the *Radius* : to the *Co-sine* of one of the given *Latitudes* : : So is the *Co-sine* of the other given *Latitude* : to a 4th *Sine*.

As the 4th *Sine* : to the *Sine* of Half the Sum of the Three *Sides* : : So is the *Sine* of Half the Sum of the Three *Sides* lessen'd by the Distance : to a 7th *Sine*. To which if you add the *Radius*, Half that Sum will be the *Co-sine* of Half the Difference of *Longitude* required.

Fig. 37. CASE 4th. If the Latitudes of both Places were given, and the Angle of Position 'twixt one and the other, to find their Difference of Longitude; then, in the Spherical Triangle *LpV* are given *pL* the Latitude of one of the Places, suppose *London*; and *pV* the Latitude of the other Place suppose *Vienna*: and the Angle at *L* the Angle of Position 'twixt *London* and *Vienna*: Whence to find the Angle *p*, their Difference of Longitude, it will be, by the known Laws of *Spherical Triangles*,

$$R : c.S.pL :: T.\angle L : c.T.\ 4\text{th } Arch.$$

Again,

$$T.Vp : T.Lp :: c.S.\ 4\text{th } Arch : c.S.\ 5\text{th } Arch;$$

Or as

$$c.T.\ Lp : c.T.Vp :: c.S.\ 4\text{th } Arch : c.S.\ 5\text{th } Arch.$$

If the Angle *L* be Obtuse, the 5th *Arch* — 4th *Arch*; if Acute, the 5th *Arch* + 4th *Arch* gives the Difference of Longitude.

CASE

CASE 5th. If the Latitude of one Place, suppose of *London*, its Distance from another, suppose *Vienna*, and Angle of Position 'twixt *London* and *Vienna*, be given, to find their Difference of Longitude, and Inclinations of the Plains of the Meridians passing through the two Places: Then, in the Spherical Triangle pLV are given $pL \angle L$, and pV, *viz*, two Sides, and the contain'd Angle: Whence to find $\angle p$ it will be, Fig. 37.

$R : c.S. \angle p :: T. LV : T.$ 4th *Arch.*

4th *Arch* + pL. if the Angle at L be Obtuse: If Acute, pL—4th *Arch* gives a 5th *Arch*.

Again,

$S.$ 5th *Arch* : $S.$ 4th *Arch* :: $T. \angle L : T. \angle p.$

If the Latitude be requir'd;

$c.S.$ 4th *Arch* : $c.S.$ 5th *Arch* :: $c.S.\ LV : c.S.\ pV$, or to the Sine of the Latitude.

SECT. II.

THE Rules deliver'd in the latter part of the former Section, for finding the *Difference* of *Longitude* or *Meridians* of Places, the only thing wanting to render the Sciences of *Geography* and *Navigation* perfect, being not so practicable as could be wish'd for; we shall, in this Section, give an Account of some of the principal Methods that have been proposed by the modern *Astronomers*: Which Methods being rigidly true, and easie to be learned, wou'd (if put in practise by those Gentlemen whose Inclinations or necessary Affairs oblige them to travel into most Parts of the known World) soon afford us a much

more ample and true Deſcription of our Earth, than has ever yet appear'd Abroad.

And tho, it muſt be confeſſed, that even theſe Methods (moſt of which have been made uſe of with very good ſucceſs, in determining the Difference of Longitudes between ſome of the chief Cities and principal Head-lands in *Europe*, and other known Parts of the Habitable World) cannot be made uſe of at Sea, by reaſon of the unequal Motion of a Ship; yet if our Sea-men would accuſtom themſelves to them, by making uſe of them in the ſeveral Ports to which they ſail, nothing would contribute more to removing the Imperfections of our *Geography*, and determining thence the true Place of a Ship at Sea, unleſs ſome lucky Perſon could hit upon an Expedient to render them equally uſeful upon the Ocean, as upon the *Terra Firma*.

But for the better underſtanding the ſeveral Methods, it is neceſſary to premiſe, That the *Diffe ence* of *Longitude* or *Arch* of the *Æquator* intercepted between the two *Meridians* paſſing through the two Places, is analogous to the Quantity of Time that the *Sun* requires to move from the *Meridian* of one Place to that of another, or in the Language of the *Copernicans*, that is elapſed between the Application of the *Meridian* of one of the Places to the *Sun*, and the *Meridian* of the other. For ſince the *Sun* finiſhes its Diurnal Motion in the Space of One Natural Day, or 24 Hours, or which is the ſame, the Revolution of the *Earth* about her *Axis*, is perform'd in the ſame Time, it follows, that in every Hour there paſſes over $\frac{1}{24}$ of the *Meridian* 360 *Degrees*, or 15 *Degrees*: Whence it is, that if the *Difference* of *Longitude* 'twixt two Places is known, the *Difference* of *Time* 'twixt the two Places is known alſo; and conſequently,

ſequently, the *Hour* being known in one Place, the *Hour* in the other is alſo known; and, *vice verſa*, if the *Differance* of *Time* in two Places is known, the *Difference* of *Longitude* is alſo known, by reducing the Time into *Degrees*, allowing 15 *Degrees* to an *Hour*.

Hence it is, that if in two or more Places the Hour be the ſame, thoſe Places lie under the ſame *Meridian*, and have therefore no *Difference* of *Longitude*.

If there be a *Difference* of *Times*, that Place in which the Time is the lateſt lies to the *Eaſtward* of the two; or rather, that Place in which the *Sun* is fartheſt diſtant from the *Meridian*, if you reckon the Day from Noon to Noon, or from the Time of the *Sun*'s tranſiting the *Meridian* to the Time of his next tranſit over it again.

Whence, if any two Perſons can view the ſame *Phænomenon* at once, the Difference of the Times of Obſervation in each Place, or the Differences of the Diſtance of the *Sun* from each of the *Meridians* will be the Difference of *Longitudes* of thoſe two Places.

Now ſince an *Eclipſe* of the *Moon* proceeds from nothing elſe than an Interpoſition of the *Earth* 'twixt her and the *Sun*, the Moment that any part of her Body is deprived of the Solar Rays, it is viſible to all thoſe People who can ſee her at the ſame time. Whence if two or more People, in two or more different Places, note but the Times when it firſt Began or Ended, or note the Time when any Number of Digits was *Eclipſed*, or when the Shadow began to touch or cover any *Notable Spot*, the Difference of thoſe *Times*, when compared together, will give the Difference of *Longitude* between the *Places* of Obſervation: And of theſe we have various Inſtances.

On 21 *December*, 1675, at 16 *h.* 07 *m.* 15 *sec.* Mr. *Flamsted*, at the *Royal Observatory* at *Greenwich*, Observed the End of a *Lunar Eclipse*; and Mr. *Cassini* Observ'd, That the End of the same *Eclipse* happen'd at the *Royal Observatory* at *Paris* at 16 *h.* 15 *m.* 25 *sec.* Whence the Difference of *Times* 'twixt the two Places of Observation is 8 *m.* 10 *sec.* equal to 2 *deg.* 2 *m.* 30 *sec.* Whence since the *Sun* was removed farther off from the *Meridian* of *Paris*, than from the *Meridian* of the *Observatory* at *Greenwich*; it follows, that *Paris* is to the *Eastward* of *Greenwich* 2° 2′ 30″ equal to the Difference of *Longitude* between those two Places. Again,

Mr. *Flamsted*, on the 11th of *February*, 168½. Observed the Beginning of a *Lunar Eclipse* at 9 *h.* 12 *m.* ½. the Beginning of the same *Eclipse* was Observed at *Lisbon* in *Portugal* to happen at 8 *h.* 31 *m.* *Post Merid.* Whence the Difference of *Meridians*, or Difference of *Longitude* 'twixt *Greenwich* and *Lisbon* is 41 *min.* ½. of *Time*, or 10 *deg.* 22 *m* ½. *Lisbon* lying so much to the *Westward* of *Greenwich*.

Now by comparing the Difference of *Longitude* 'twixt *Paris* and *Greenwich*, and *Greenwich* and *Lisbon*, may the Difference of *Longitudes* 'twixt *Lisbon* and *Paris* be determin'd.

Greenwich lies to the *Westward* of *Paris* 2° 2′ 30″. *Lisbon* lies to the *Westward* of *Greenwich* 10° 22′ 30″. Whence *Paris* lies to the *Eastward* of *Lisbon* 12° 25′ 00.

The *Longitudes* of Places may also be determin'd from the Observations of *Solar Eclipses*; but these being incumber'd with the Consideration of *Parallaxes*, are not so proper as those of *Lunar Eclipses*: However, any One who is Curious may see the Difference of *Longitude* of some Places

Places, deduced from *Solar Eclipſes*, by Mr. *Caſſini*, *Philoſ. Tranſact.* N° 5. p. 126.

Theſe *Eclipſes* happening but rarely, another Expedient has been thought of, *viz.* the *Eclipſes* of the *Satellites* of *Jupiter*.

Jupiter has been found to have Four *Satallites* or *Moons* conſtantly attending him, always obſerving the ſame Laws in their Revolutions about him: The firſt or neareſt to his Body finiſhes his Revolution about him in the Space of 1 Day, 18 Hours, 28 Minutes; the ſecond, in 3 Days, 13 Hours, 17 Minutes; the third in 7 Days, 3 Hours, 59 Minutes; the fourth, in 16 Days, 18 Hours, 5 Minutes.

Now it having been found, that neither *Jupiter*, nor his Attendants have any Native Light of their own, but ſhine with a boow'd Light from the *Sun*, it happens that each of theſe, in their Revolution round *Jupiter* ſuffer two *Eclipſes*, one at their Entrance into the *Shadow*, the other at the Entrance of their Paſſage behind the Body; whence it comes to paſs, that in each Revolution of the *Satellite* there are Four remarkable Obſervations, by any one of which the Buſineſs may be done, *viz.* one at the Entrance into the Shadow, and one at the Emerſion out of it; one at the Entrance behind the Body, and another at the Coming out; but the latter of theſe, *viz.* the Entrance behind the Body and Time of Coming out is not ſo much regarded by *Aſtronomers*, as the Immerſion and Emerſion into and out of the Shadow; becauſe in the former, the difficulty of pronouncing the exact Time is very great, requiring good Eyes, and large Teleſcopes; but the latter is eaſie, becauſe the ſwift Motion of the *Satellites* plung'd them ſo ſoon into the Shadow that it is no difficult matter to pronounce, with any Teleſcope, by which they may be ſeen, the

the exact Time of their Immersion and Emersion.

Now, inasmuch as each of these happens at the same Moment of absolute Time, if two or more Persons, in different Places note the Times, these, compared together, will shew the Difference of *Longitudes* of the Places of Observation.

Among the many Observations that have been made, by which the *Longitudes* of many Places have been determin'd, we shall extract some out of the *Philos. Transact.* N° 177.

Anno 1680. *Oct.* 23. S. V. S. *Jos. Ponthio* and *Marco Antonio Cellio* Observ'd the total Immersion of the first *Satellite* into *Jupiter's* Shadow, at *Rome*, at 10 *h.* 7 *m.* 53 *sec. Post Merid.* Which was also Observ'd by Mr. *Flamsted*, at the *Royal Observatory* at *Greenwich*, at 9 *h.* 15 *m.* 41 *seconds.* Whence the Difference of *Meridians* 'twixt *Rome* and *Greenwich*, is 52 *m.* 12 *sec.* in *Time*, answering to 13° 03′ *Rome* lying so much to the *Eastward* of *Greenwich*.

In the former Instance, we have suppos'd the same Observation to be made by two different Persons in different Places; but the Motion of the *Satellites* being truly settled, there is need only of a Catalogue of the *Eclipses* to be publish'd for the *Meridian* of any one Place, and the Observation made in different Places, compar'd with the Times set down in the Catalogue, will give the Difference of *Longitude* 'twixt the Places of Observation, and the Place for which the Catalogue is calculated.

And as an Instance of the Exactness of the Tables for the first *Satellite*, I need only mention an *Observation* of *Francis Blanchini*, made at *Rome* in the Year 1685, when, on the 28th of *January*, he observ'd the total Immersion of the first

first *Satellite* at 11 *h.* 19 *m.* $\frac{1}{4}$; which, by the Tables, happened at *Greenwich* at 10 *h.* 27 *m.* $\frac{1}{4}$: Whence the Difference of *Meridians* 'twixt *Rome* and the *Observatory* at *Greenwich* is 52 *m.* $\frac{1}{2}$, agreeing very well with the former Deduction.

When we consider the great Number of these *Eclipses* that happen every Year, there being scarce a clear Night when *Jupiter* may be seen, but an *Eclipse* of one or other happens, the Easiness with which they may be made, there being requisite only a *Time Keeper*, and a *Telescope* of 8 Foot long, it is much to be wonder'd that they are so neglected by our Sea-men, and so few of the Remarkable Places to which we Traffick are not better determined.

If any one is desirous of having a more ample Account of the Use of these *Satellites* in Determining the Difference of *Longitudes* of Places, let him consult the *Philos. Transact.* N° 151, 154, and 165.

Besides these, there is another Method equally Useful, Expeditious, and Certain, and that is, the *Appulses* of the *Moon* to certain *Fix'd Stars*, and their *Occultations* by the Interposition of her Body, For the *Moon* finishing her *Revolution* in her *Orb* in the Space of 27 *days*, 7 *h.* 43 *m.* there are but few clear Nights when the *Moon* does not pass over or so near to some *Fix'd Star*, that her Distance from it, or the Time of her Visible Conjunction with it, may be easily observ'd by the *Telescope* and *Micrometer* only; and these when observ'd in Different Places, and compared together, or with the Visible Times computed to the *Meridian* of some Place, when a good *Theory* of the *Moon* shall be obtained, will shew the Difference of *Longitudes* of those Places. An Instance of the Use of this Method we have in the Determination of the Difference of

of *Longitude* between *London* and *Ballasore* in *India*, as follows.

On the 28th of *October*, 1680. at 8 *h.* 6 *m.* an Immersion of the *Bull's Eye* was Observ'd at *London*, when the true Place of the *Moon* corrected by Parallax was ♊ 4° 32′ 24″; but at *Ballasore Road* (in the Latitude of 21° 20′ *N.* and about 20 Miles *E. S. E.* from the Town) the true Place of the *Moon* was ♊ 5° 54′, that is, 1° 21′ 36″ more than at 8 *h.* 6 *m.* at *London.* Now, according to the *Moon's* Velocity at that Time, she passeth an *Arch* of 1° 21′ 36″ in 2 *h.* 8. *m* 40 *sec.* of *Time.* So then, at 10 *h.* 14 *m.* 40 *sec.* at *London.* the *Moon* was in the same Place as at 16 *h.* 00 *m.* at *Ballasore-Road*: Whence the Difference of *Longitude* will be 5 *h.* 45 *m.* 20 *sec.* or 86° 20′ *Ballasore-Road* being so much to the *Eastward* of *London.*

Again on the 22d of *December*, 1680. the Immersion of the *Bull's Eye* was found by Calculation, to be at 14. *h* 49. *m.* at *Ballasore*, the *Moon's* true Place was ♊ 6° 30′ 30″; and at 7 *h.* 46 *m.* 12 *sec.* the correct Time of the Immersion at *Dantzick*, the true Place was ♊ 4° 55′ 11″, that is, 1° 35′ 20″ short of the Place deduc'd from the *Observation* at *Ballasore-Road*, which makes in *Time* 2 *h.* 32 *m.* 40 *sec.* Whence it follows, that 10 *h.* 18 *m.* 52 *sec.* at *Dantzick* makes 14. *h.* 49 *m.* at *Ballasore-Road*, and the Difference of *Longitude* 4 *h.* 30 *m.* 8 *sec.* and *Dantzick* being 1 *h.* 15 *m.* 30 *sec.* more *Easterly* than *London*, *Ballasore-Road* will be from *London* 5 *h.* 45 *m.* 38 *sec.* or 86° 24′.

For a further Confirmation hereof, Mr. *Benjamin Harris* being a-shore at *Ballasore-Town*, he Observed with very great Care and Exactness, *November* the 18th, 1680. that at 9 *h.* 13 *m.* the *Star* which *Tycho* calls, *in Cotyla dextra Aquarii*

duarum

duarum præcedens, (and which was then in *Aquarius* 28° 52′, and *Latitude* 2° 46′ *North*) was in a Right Line with the Cusps of the *Moon*, then near the 1st Quarter the *Moon*'s Place at that time, by the Horroccian Theory, *viz.* at 2 *h.* 53 *m.* at *London* is found ♒ 29° 22′ 10″; but at 9 *h.* 13 *m.* at *Ballasore* her Place was in ♒ 29° 41′ 17″, that is 19′ 7″ more than at *London*, which in *Time* gives 36 *m.* So that 3 *h.* 29 *m.* at *London*, was 9 *h.* 13 *m.* at *Ballasore*; and the Difference of *Longitude* therefore 5 *h.* 44 *m.* or 86° 00′ precisely agreeing with the former Deductions. Vid. *Philos. Coll.* N° 5.

The same may be done from the *Observations* of the Occultations of the *Superior Planets* by the Interposition of the *Sun* and *Moon*, or by the Transits of the *Inferior Planets* over the *Sun's* Body, and by an Obscuration of this Kind, Monsieur *Cassini* pronounced the Difference of *Longitude* between *Canton* in *China* and *Paris* to be 7 *h.* 23 *m.* equal to 110° 45′ from an Emersion of *Mercury* from the *Disk* of the *Sun* Observed at *Canton* and *Noremberg*, and an *Eclipse* of the *Moon* Observ'd at *Noremberg* and *Paris*.

In like manner, from an *Observation* of the Occultation of *Mars* by the *Moon*, the 21st of *August*, 1676. Observed at *Dantzick* and *Greenwich*, the Difference of *Meridians* 'twixt *Greenwich* and *Dantzick* is found to be 1 *h.* 14 *m.* 49 *sec.* in *Time*, answering to 18° 41′ 15″.

Any One that is desirous to have a more ample Account of the manner of Determining the *Difference* of *Longitudes* of Places, either by the *Observations* of the *Eclipses* of the *Sun* and *Moon*, or by the Occultation of the *Fix'd Stars*, as also of the Method of Calculating the

Beginning, Middle, End, and Duration of any one of these, Appearances, let him consult *A Treatise of Astronomy*, Written by Mr. *Flamstead*, and Printed in Sir *Jonas Moor*'s First Volume of the *Mathematicks*.

And though this latter Method labours at present under some Inconveniency, by the Imperfection of the *Lunar Theory*; yet so good a *Foundation* has been laid, at the Royal *Observatory* at *Greenwich*, for removing the Difficulties, that the World may soon hope to see a more Correct *Theory* than has yet appeared Abroad.

A much more easy and not less certain Method than any of these, was that propos'd by Monsieur *Huygens*, by the help of *Pendulum Watches*, (a large Account of the Manner how they are to be used, may be seen in *Philos. Trans.* N° 47.) yet even this, tho practised with some Success by Major *Holmes*, in a Voyage from the *Isle of St. Thomas* under the Line, to the *Isle of Fuego* one of the Isles of *Cape Verd*, labours under some Difficulties as are not easily to be removed, as is known to every One who is well acquainted with their Mechanism.

Several other Methods have been proposed, for finding the Difference of *Longitudes* of Places, but the chief of them depending upon a perfect Knowledge of the *Theory* of the *Planets*, cannot be put in practice with any desirable Success, till we are better acquainted with the Motions of the Heavenly Bodies.

In

In the mean time, it could be wiſh'd our Seamen, who will be the principal Gainers by it, whenever it ſhall be rendered more Practicable, would omit no Opportunity, when they ſhall be in Harbour, of making ſuch *Obſervations* as offer, for Correcting the Sea-Coaſts; ſince it is of little Service to know the exact Place of a Ship at Sea, if the Situation of the Place to which ſhe is bound be not exactly known alſo.

FINIS.

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Fig. 14.

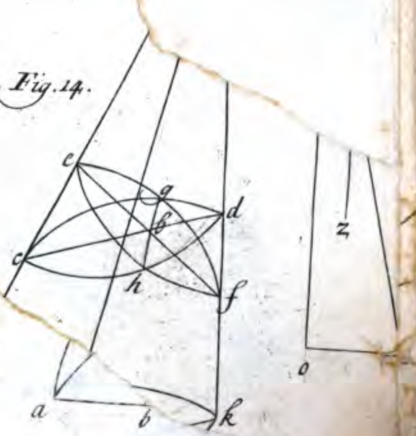

Fig 4.


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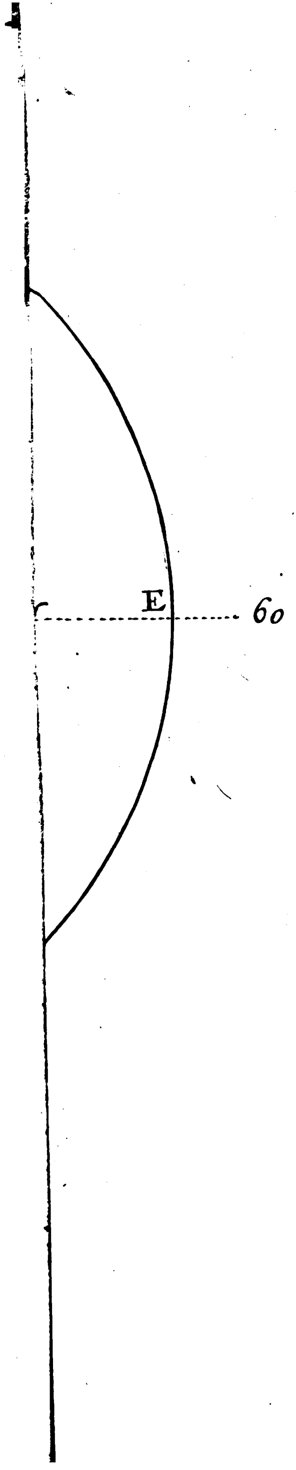
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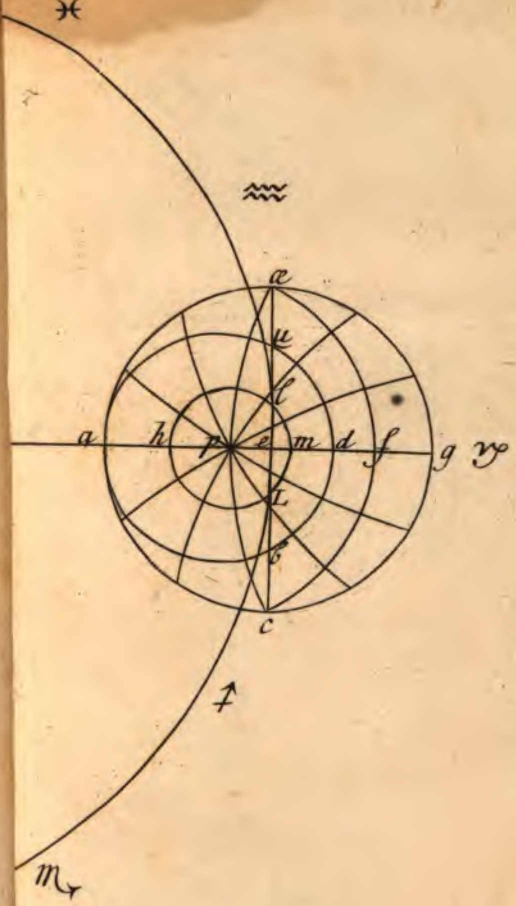
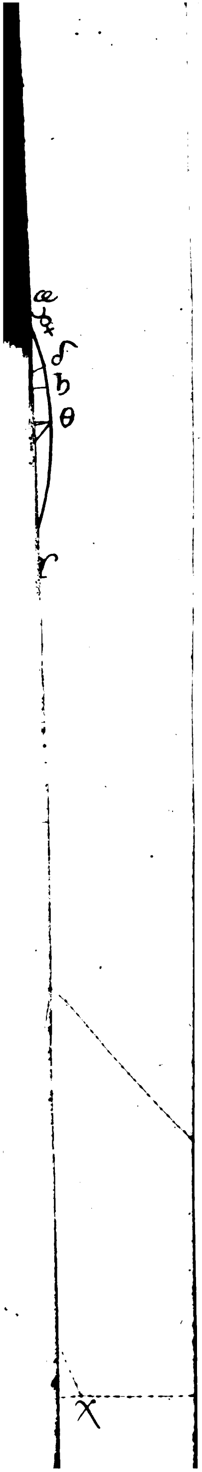
Fig: 31.

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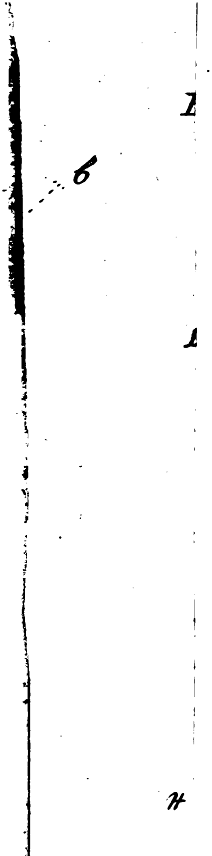

Fig. 39